

# A Study of Revenue Flows in Packet Networks under Multiple Administrative Domains

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## Abstract

*The exponential growth of the Internet in virtually every metric has sparked a discussion about the role of pricing in public data networks. Initial research results demonstrate that priority pricing, congestion pricing, and other pricing schemes have the potential for improving social welfare over flat-price schemes. While promising, these initial results have significant shortcomings that have limited their applicability; notably that they usually only consider a single network and always consider a single administrative domain when they include multiple switches. Furthermore, most prior analyses usually focus either on the technical or economical aspects without consideration of the interplay between the two. This paper addresses the need for a detailed study of more realistic networks under multiple administrative domains. The interactions between networks under multiple administrative domains are analyzed in terms of revenues collected by these networks. More specifically, we study the revenue flows or settlements among multiple interconnected networks when one of the networks changes its technical or economic strategies.*

**Keywords.** Packet Networks, Network Pricing, Multiple Networks, Simulation, Modeling, Settlements.

## 1 Introduction

Prices can be used to control network traffic by giving signals to users to modulate their traffic when the network is congested. This has been shown previously by several researchers, both analytically and using simulation [1, 2, 6, 7, 9, 10]. While this result is not surprising to most economists, a variety of technical problems must be addressed to understand and implement price-based resource allocation schemes.

Most studies of pricing in computer networks con-

sider only a network under a single administrative entity. Lehr and Weiss [4] argue that new economic and technical issues arise when multiple, administratively independent networks are interconnected that are not readily captured in the economic studies of single networks. To support this argument, they developed a static model that extended earlier work by Mackie-Mason and Varian [5]. In this work, they assumed that both networks used a congestion pricing scheme.

To address the above issues, we have developed a general simulation-based framework to capture the essential properties of pricing in computer networks. This framework is designed so that it can easily accommodate different user models, pricing models, and network models. The user model supports a wide variety of user profiles, including interactive users, bulk users, and real-time users. Traffic from each user model is characterized by its burstiness, the length of its packets and a desired value of end-to-end network delay. Interactive users generate streams of short packets with moderate end-to-end delay requirements. Bulk users are characterized by large size packets and a high tolerance for long end-to-end delays. Real-time users, on the other hand, are characterized by stringent end-to-end delay requirements and large size packets, such as video packets. Depending on the user model, a decay function is associated with each packet. The decay function models the user's loss in value as a function of time.

The network model can be tailored to represent either a network administered by a single network service provider or multiple networks administered by multiple network service providers. Both models are of interest: the former to evaluate the operation of a network under a single administrative domain and the latter to develop a richer understanding of the interactions between multiple networks under multiple administrative domains. For this paper, we will focus on the latter; the question of single network service

provider was addressed in [8].

The pricing models considered in this study are flat pricing and congestion pricing. We consider specifically flat pricing schemes at various price levels as well as a congestion-based pricing scheme. Using flat pricing, users are charged a fee for every unit of traffic they generate. Congestion pricing, on the other hand, is responsive to traffic load. Congestion prices rise as the demand for the network resources increases. Network costs or fixed charges that users may experience as part of a two-part tariff are not considered. Flat pricing and congestion pricing strategies are implemented using route dependent pricing scheme.

We use the above framework to simulate networks under multiple administrative domains. In the case of multiple administrative domains, any network in a network-of-networks can have its own technical and economic strategies. Also, each network can optimize profits and technical operations locally. Many modern network structures consist of networks connected in serial fashion, with each network in the chain being separately administered and operated. A network service provider may want to change the technical and economic strategies of its network in order to gain profits. A network can change its technical strategy by changing its capacity and traffic load. A network can change its economic strategy by changing its pricing schemes and price levels. When a network changes its technical and economic strategies, it affects the traffic and revenue of other networks.

In this paper, we study the behavior of other networks if one of the networks changes its technical and economic strategies. The incentives of a network to change its behavior and the resulting consequences of this behavior on other networks are analyzed in terms of revenue flows or settlements among the networks. The other networks may be directly connected to the network changing its strategies or they may be  $n$  hops away. The other networks can also be either mixed or transit only networks. A mixed network generates revenue through originating, terminating, and transit traffic. A transit only network collects revenue by transmitting the traffic of other networks only because it does not have any originating or terminating traffic.

The rest of the paper is organized as follows. Section 2 describes user model, network models, and pricing models. Section 3 talks about experimental design. Section 4 presents and analyzes the results obtained for the multiple networks. Section 5 discusses and concludes the paper. Section 6 introduces our future work.

## 2 Model Description

The results that we discuss in subsequent sections depend heavily on the way in which the model is formulated. In this section, user model, network models,

and pricing models are described.

This model is implemented using CSIM simulation package [3]. CSIM is a process-oriented discrete-event simulation package based on C programming language.

### 2.1 User Model

An important part of the model is the way in which the user is modeled. Each user  $i$ , or  $U_i$  is characterized by its task and behavior as:  $\{U_i : Tk_{ij}, B_i\}$ , where  $Tk_{ij}$  is task  $j$  executed by user  $i$  and  $B_i$  is the behavior of user  $i$ .

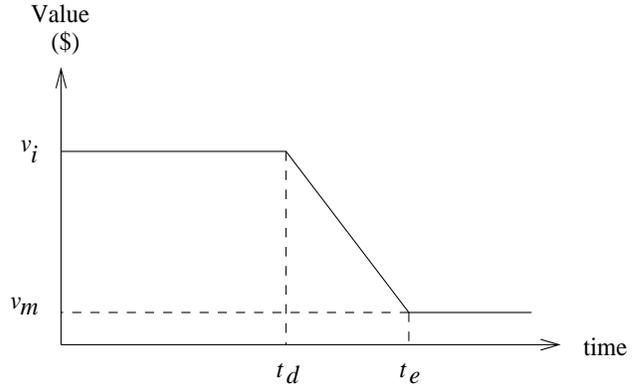


Figure 1: Decay function for Packets

Users execute tasks, which generate packets. Each user performs a task of certain type. A task  $Tk_{ij}$  is characterized by its type and value as:  $\{Tk_{ij} : Tp_{ij}, V_{ij}^t\}$ , where  $Tp_{ij}$  is type of task  $j$  of user  $i$  and  $V_{ij}^t$  is the time  $t$  dependent value of task  $j$  of user  $i$ . The value of a task is the sum of the values of the packets associated with that task. At the start of the task, the same value is assigned to all packets of the task. The value of each packet decays with time. The user uses a *decay function* related to the type of task that models the user's loss in value as a function of time. Decay functions are of the form shown in Figure 1. Each packet associated with a task type receives the identical profile and gets the full allotment of time. Note that value figures are relative, so that  $v_i = 1$  and  $v_m = \beta v_i$ , ( $0 \leq \beta \leq 1$ ). A "hard deadline" means that  $t_e \approx t_d$ , and more moderate deadlines imply a larger difference between  $t_e$  and  $t_d$ .

Users have a behavior associated with them. The behavior  $B_i$  of a user  $i$  depends on the values of packets it sends, and the price and delay information received from the network. The user compares the value of a packet with the price for sending the packet. If the value of the packet is greater than the price, the user sends the packet into the network. Otherwise, the user does not send the packet and waits until the next price interval for new price update. The user can also record

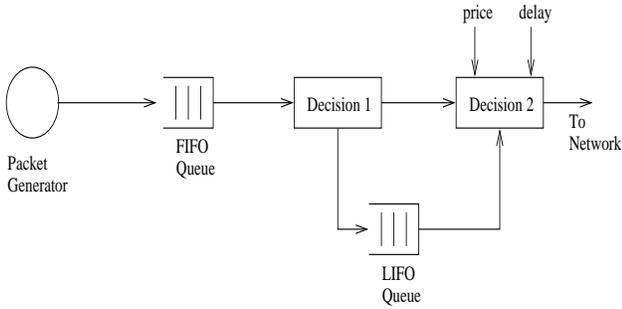


Figure 2: User Model

the experienced delay,  $d_t$ , of the most recently transmitted packet. The user computes  $\hat{d}_{t+1}$ , the delay estimate for the next packet to be transmitted. The user does this by computing  $\hat{d}_{t+1} = (1 - \alpha)d_t + \alpha \hat{d}_t$ , where  $0 \leq \alpha \leq 1$  is a weighting factor and  $\hat{d}_t$  is the delay estimate for the most recently transmitted packet. The user uses this delay information in computing the value of the packet according to the decay function.

The tasks can be of different types. In this model, we assume three types of tasks: interactive, real time, and bulk. In each case, the packet length is computed by  $n_i l$ ,  $n_i = 1, 2, \dots$ . These are characterized as follows: **interactive**, real time, and bulk. In each case, the packet length is computed by  $n_i l$ ,  $n_i = 1, 2, \dots$ . We characterize these as follows: **Interactive users** (e.g., telnet or WWW) generate a stream of short packets ( $n_i = 1$ ) with moderate deadlines (no packet loss); **real time users** (e.g., voice, video) are characterized by large packets ( $n_i = 3$ ) and a hard deadline (packet loss permitted); and **bulk users** (e.g., ftp) consist of large packets ( $n_i = 3$ ) with long deadlines (no packet loss).

Operationally, the process works as follows: (1) Values are assigned to packets when a task initiates; these values are randomly drawn from a uniform distribution (0,1]. Each packet associated with a task is given this *nominal value*,  $v_i$ . (2) If the packet is transmitted prior to the deadline ( $t_d$ ), its value remains intact. This is implemented by placing the packet first in a FIFO (First In First Out) queue in the user. (3) If the packet resides in the buffer too long, its deadline is exceeded, and the value begins to decay linearly until the expiration ( $t_e$ ). This is implemented by placing the packets that have waited in the FIFO beyond  $t_d$  into a LIFO (Last In First Out) queue (Decision 1 in Figure 2). (4) The user will always transmit the FIFO packets first, then the LIFO packets, up to either capacity of the network or the point at which the price exceeds the value of the packet, whichever comes first (Decision 2 in Figure 2). This ensures that the highest value packets are transmitted first.

## 2.2 Network Models

The network model consists of three packet switches connected in series by communication links. Each packet switch is modeled as a single queuing system. This model does not place a limitation on buffer size (so that there can be no dropped packets once a packet enters the network), and each buffer is serviced as a FIFO. The service rate is limited by the bandwidth of the link to the next destination. For this simulation, the channel capacity is X packets/second. Thus, the actual delay that a packet experiences is composed of the sum of waiting time (queuing delay) and transmission delay. Transmission delay is faced by all packets, so the primary source of delay variation is queuing delay. It is therefore the queuing delay that is the focus of our interest.

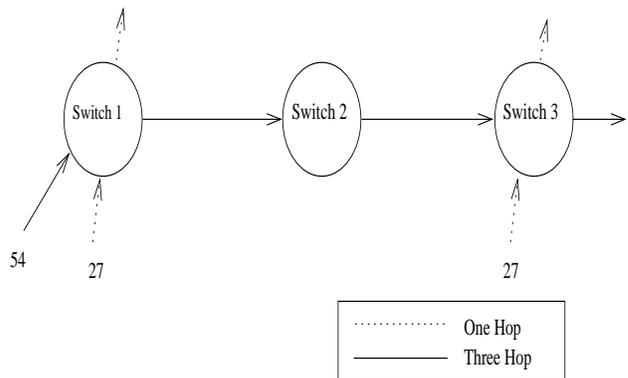


Figure 3: Network Model - Switch 2 is Transit Only

We chose to model 108 users so that any user's decision would have a negligible impact on the price or performance faced by other users. It is also assumed that the service providers face a contestable market for network services which prevents the network service provider from charging monopoly prices.

The network models for multiple service providers are shown in Figures 3 and 4. In Figure 3, the traffic is arranged such that the middle switch neither originates nor terminates any traffic. The middle switch just transits the traffic of other switches. This implies that its only source of revenues comes from congestion fees. In Figure 4, the traffic is arranged such that the middle switch has mixed traffic consisting of originating, terminating, and transit traffic.

The networks also inform each user of the delay that the most recently transmitted packet experienced. The user uses this delay value  $d_t$  to compute the estimated delay  $\hat{d}_{t+1}$  as described in section 2.1. While networks do not provide delay information to the user in practice, it is assumed that a user could gather this information at virtually no cost.

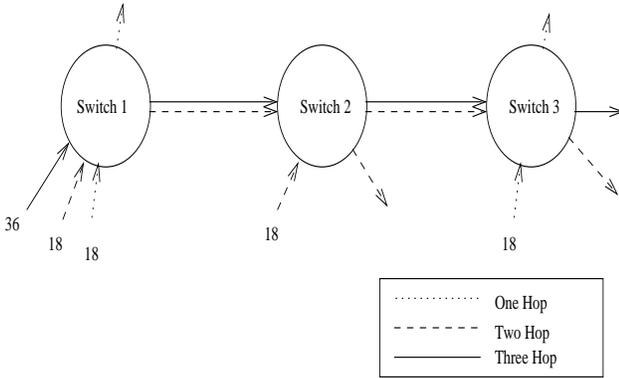


Figure 4: Network Model - Switch 2 is Mixed

## 2.3 Pricing Models

We use two pricing models or strategies: flat pricing and congestion pricing. In flat pricing, the network sets a fixed per packet price ( $0 \leq P \leq 1$ ) that is faced by all packets. Non-usage sensitive prices, such as attachment prices are not included in our model. In congestion pricing, each switch  $i$  computes a price  $p_i = a + b \left[ \frac{\rho_i}{1 - \rho_i} \right]$ , where  $\rho_i$  is the utilization of switch  $i$  in a price interval, and  $a$  and  $b$  are constants that are selected so that the price is in the range of packet values ( $0 \leq p_i \leq 1$ ). We make no assertion here that this is an optimal congestion price; however, we do assert that this price behaves in a way that is similar to the way in which optimal congestion prices are likely to behave.

In case of multiple service providers, each can have its own pricing scheme. For example, one might use flat pricing while others can use congestion pricing schemes. Flat and congestion pricing strategies are implemented using route dependent pricing schemes. In a route dependent pricing scheme, the price faced by users is the sum of the prices of the switches through which the users' packets are transmitted. The overall network price  $P$  is computed as:  $P = \sum_{i=1}^n p_i$ , where  $p_i$  is the price of switch  $i$ .

From an economic perspective, time is divided into fixed "price intervals". At the beginning of each price interval, the network announces a per-packet price that is valid for the duration of the price interval. For flat prices, this computation is simple; the network simply sets price and all users pay the same rate or price depending on the route of their packets.

For congestion prices, each switch  $i$  in the network computes its utilization  $\rho_t^i$  as  $\rho_t^i = \lambda_t^i / \mu_t^i$ , where  $\rho_t^i$  is the utilization of switch  $i$  during price interval  $t$ ,  $\lambda_t^i$  is the arrival rate of packets at switch  $i$  during price interval  $t$  and  $\mu_t^i$  is the service rate of packets at switch  $i$  during price interval  $t$ . Each switch  $i$  in the network computes  $\hat{\rho}_{t+1}^i$ , the utilization estimate for the next price interval  $t + 1$ . The switch does this by comput-

ing  $\hat{\rho}_{t+1}^i = (1 - \alpha)\rho_t^i + \alpha\hat{\rho}_t^i$ , where  $0 \leq \alpha \leq 1$  is a weighting factor and  $\hat{\rho}_t^i$  is the utilization estimate of switch  $i$  for the current price interval  $t$ . Each switch computes a price  $p_{t+1}^i$  for the next price interval  $t + 1$  as  $p_{t+1}^i = a + b \left[ \frac{\rho_{t+1}^i}{1 - \rho_{t+1}^i} \right]$ . The overall network price  $P_t$  is computed based on the individual switch prices as  $P_t = \sum_{i=1}^n p_{t+1}^i$ . The network also computes  $\hat{P}_{t+1}$ , the price estimate for the next price interval  $t + 1$ . The network does this by computing  $\hat{P}_{t+1} = (1 - \alpha)P_t + \alpha\hat{P}_t$ , where  $0 \leq \alpha \leq 1$  is a weighting factor and  $\hat{P}_t$  is the price estimate for the current price interval  $t$ . The network announces this estimated price  $\hat{P}_{t+1}$  to the users. The users use the estimated price in making a decision by comparing the estimated price with the value of the packets that they want to transmit.

Revenue for each switch  $i$  is computed as the product of switch price  $p_i$  and the number of packets transmitted by switch  $i$  during a price interval. Each switch  $i$  collects revenue for packets transmitted through it and aggregates its revenue over the simulation time. In case of multiple service providers, each switch collects its own revenue independently of other switches.

## 3 Experimental Design

We conducted a number of experiments on multiple networks with each network being under a multiple administrative domain. Each of the switches in Figures 3 and 4 are considered to be under a separate service provider. There are three independent network service providers administering each of the individual networks separately.

In this study, multiple networks are supposed to be heterogeneous and multiply-administered computer networks. Heterogeneous means that any network in a network-of-networks can have its own technical and pricing strategies. Multiply-administered means that each network optimizes profits and technical operations locally. Many modern network structures consist of networks connected in serial fashion, with each network in the chain being separately administered and operated. A network service provider may want to change the technical and pricing strategies of its network in order to gain profits. In such an environment, the revenue flows or settlements among networks in a network-of-networks becomes an interesting issue. A network changes its behavior by changing its technical and pricing strategies. We study the effects due to the changing behavior of a network on other networks which are directly or indirectly connected to the network which is changing its behavior. The effects produced due to the changing behavior of networks are analyzed in terms of revenue flows.

A network can change its behavior as follows:

**Change Pricing Scheme:** Each network can have

a pricing scheme of its own. For example, one network can use flat pricing scheme whereas the other networks can use congestion pricing schemes. Each network can also change the algorithm or formula for computing the prices. Changing pricing schemes is a strategic decision. Three networks with two pricing schemes can have eight possible combinations. The four meaningful combinations considered here are: Flat-Flat-Flat (FFF), Flat-Congestion-Flat (FCF), Congestion-Flat-Congestion (CFC), and Congestion-Congestion-Congestion (CCC).

**Change Flat Price:** This applies only to those networks that use flat pricing schemes. A network can change the levels of its flat prices. The levels of flat prices of one or more networks are kept fixed whereas the levels of flat prices of other networks are varied between 0.0 and 0.9. Changing levels of flat prices is a strategic decision.

**Change Traffic:** The offered traffic to a network is varied by changing  $\rho$  for that network. The levels of  $\rho$  are: 0.1, 0.5, and 0.9.

**Change Capacity:** The capacity  $C$  of a network can also be varied. Varying capacity of a network changes utilization  $\rho$  of that network. The levels of capacity are: low (L), medium (M), and high (H). If capacity of a network is low,  $\rho$  of that network will be high for a fixed amount of traffic entering the network and vice versa.

The effects produced due to the changing behavior of a network can be seen on other networks which are in the following positions:

**Transit Only Network:** The transit only network transmits the traffic of other networks only. This kind of network does not have originating or terminating traffic. The second network shown in Figure 3 is a transit only network. Network 1 has 27 1-hop users. There are 54 3-hop users connected to network 1 whose packets go through all the three networks. Network 2 does not have any users. It just transmits the packets of network 1. Network 3 has 27 1-hop users and it transmits the packets of networks 1.

**Mixed Network:** A mixed network can have originating, terminating, and transit traffic. The second network shown in Figure 4 is a mixed network. Network 1 has 18 1-hop and 18 2-hop users. There are 36 3-hop users connected to network 1 whose packets go through all the three networks. Network 2 does not have 1-hop users. It has 18 2-hop users and it transmits the packets of network 1. Network 3 has 18 1-hop users and it transmits the packets of networks 1 and 2.

**Next Network:** The next network of a network is a directly connected network with no other networks in between the two networks. In case of three networks, next network of network 1 is network 2 and the next network of network 2 is network 3.

**Network Removed by  $n$  hops:** A network removed by  $n$  ( $n > 1$ ) hops from a network is a network which is  $n$  networks away from that network. If  $n$  is 1, the networks will become next networks to each other. In our model, the maximum value of  $n$  is 2. Network 1 is removed by 2 hops from network 3.

For the analysis of multiple networks, the mean values and 95% confidence intervals for revenues of each of the networks are computed and plotted in the next section.

## 4 Research Results

Previously, we have reported on the results of our studies of a network under a single administrative domain [8]. In this paper, we report on experiments that were conducted on multiple networks, with each network being under an independent administrative domain.

In the case of multiple networks, the revenue flows are examined more carefully. For each network, we consider the revenue sources. In the case of route dependent pricing, we can consider the revenue derived from non-local networks as being a sort of “settlements” revenue stream<sup>1</sup>.

The four meaningful combinations of networks and pricing schemes considered here are Flat-Flat-Flat (FFF), Flat-Congestion-Flat (FCF), Congestion-Flat-Congestion (CFC), and Congestion-Congestion-Congestion (CCC). Under each combination, the levels of flat price, traffic and capacity of a network are changed. The effects of these changes on transit only networks, mixed networks, next network, and networks removed by  $n$  hops are discussed in the following sections.

### 4.1 Flat - Flat - Flat (FFF)

In this case all the three networks use flat pricing schemes.

#### 4.1.1 Change Flat Price - FFF

The plots in Figures 5, 6, 7, and 8 show total revenues of networks 1, 2, and 3 versus flat prices of networks

<sup>1</sup>An interconnection arrangement between two independent service providers may be based strictly on non-traffic sensitive access charges. We do not consider such a revenue stream; only the revenue stream from usage-based charges.

2 and 3. The levels of flat prices of networks 2 and 3 are varied between 0.0 and 0.9 in all cases. The level of  $\rho$  is 0.5 in all cases.

Network 2 is a transit only network in Figures 5 and 6. The level of flat price of network 1 is fixed at 0.1 in Figure 5. The revenue curve of network 1 is maximum at  $4 \times 10^4$  and then it decreases to  $2 \times 10^4$  as the levels of flat prices of network 2 and 3 increase to 0.9. The (flat) price faced by users of network 1 also increases with the increase of flat prices of networks 2 and 3 because prices in multiple networks are computed using only route dependent pricing scheme<sup>2</sup>. As the price faced by the 3-hop users of network 1 increase, fewer packets are sent by these users which results in decreasing revenue for network 1. Network 2 is the next network of network 1. Since network 2 does not have users of its own, it only transmits the packets of network 1. The revenue of network 2 is only due the transit traffic of network 1. As the flat prices of networks 2 and 3 become 0.5 and above, network 2 cannot transmit the packets of network 1. At this point and above, the 3-hop users of network 1 will face a flat price of 1.0 (0.5+0.5) and above. At these prices, these users cannot transmit packets because the packet values ( $0 \leq \text{values} \leq 1$ ) are lower than the prices for sending the packets. Network 3 is removed by 2 hops from network 1. The revenue of network 3 is due to the its own packets plus the packets of network 1 before flat price of 0.5. After flat price of 0.5, the revenue of network 3 is due to its own packets only because the 3-hop users of network 1 can not send packets to network 3 anymore due to higher prices. The revenue of network 3 is higher than that of network 2 because network 3 has 1-hop users.

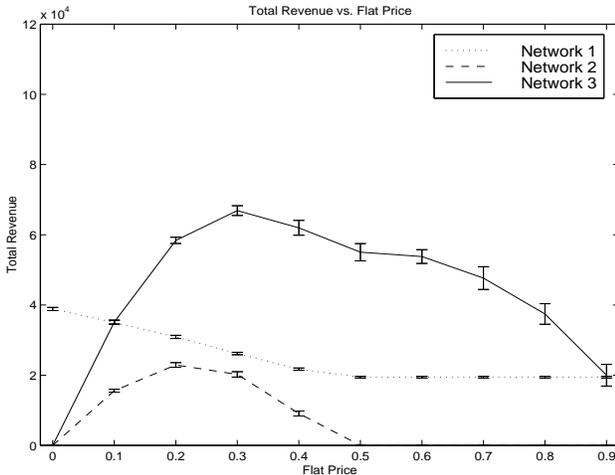


Figure 5: FFF: Total Revenue vs. Flat Prices of Networks 2 and 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.1 - Network 2 is Transit Only

<sup>2</sup>In route dependent pricing scheme, users have to pay higher price if their packets have to go through more hops.

The level of flat price of network 1 is fixed at 0.9 in Figure 6. The revenue curve for network 1 is maximum at  $4 \times 10^4$  and then it decreases quickly to  $1.8 \times 10^4$  as the levels of flat prices of network 2 and 3 approach 0.1. At this point and above, the 3-hop users of network 1 face a price of 1.1 (0.9+0.1+0.1) and above. The revenue of network 1 stays constant because only 1-hop users of network 1 send packets and they face a flat price of 0.9 all the time. The revenue of network 2 is very small before flat prices of networks 2 and 3 reach 0.1. As flat prices of networks 2 and 3 reach 0.1, the revenue of network 2 becomes zero because the users of network 1 will face a price of 1.1. Since this price is higher than the packet values, no packets are sent through network 2 by the users of network 1. The revenue of network 3 after flat price of 0.1 is due to the its own packets only. Network 3 collects revenue because the 1-hop users of network 3 can send packets all the time at all levels of flat prices. Revenue of network 3 is lower than its revenue when the flat price of network 1 is fixed at 0.1 (Figure 5) because at flat prices of 0.1 network 1 sends more packets to network 3.

Clearly, the viability of a transit only networks is substantially dependent on the behavior of the end networks. In fact, it seems that the only viable environment for a transit only network is when the (fixed) prices are low. This may explain why many transit Internet Service Providers have paired with ones that serve end users.

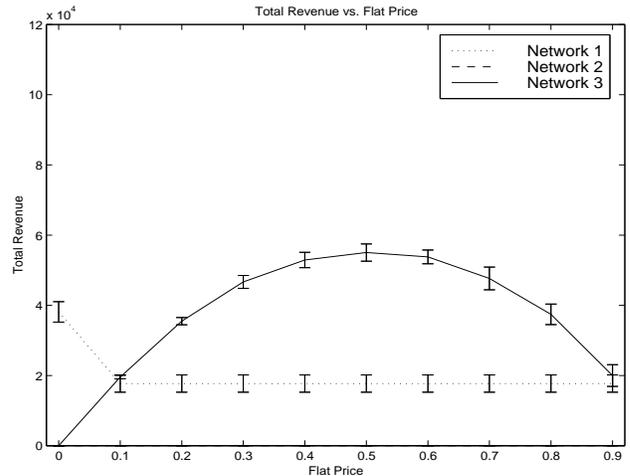


Figure 6: FFF: Total Revenue vs. Flat Prices of Networks 2 and 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.9 - Network 2 is Transit Only

The transit network service providers just provide the transport for the traffic of other networks. These providers may not generate enough revenue if the prices of the other end networks, to which the transit network is connected, are high. The providers would

like to provide other services too in order to generate more revenue. They can provide content and direct access to the users. By providing content, a network will have terminating traffic. By providing access, a network will have originating traffic. Now the network can have transit traffic as well as originating and terminating traffic. This kind of network is mixed network. We explore the results of this change by examining Network 2.

Network 2 is a mixed network in Figures 7 and 8. The level of flat price of network 1 is fixed at 0.1 in Figure 7. The revenue curves for networks 1 and 3 are almost similar to those of networks 1 and 3 when network 2 is transit only (Figure 5). The main difference is between the revenue curve of network 2 (transit only network) in Figure 5 and that of network 2 (mixed network) in Figure 7. In Figure 7, the revenue curve of network 2 (mixed network) does not become zero at the point when the flat prices of networks 2 and 3 equal 0.5. After flat price of 0.5, network 2 can still transmit the packets of network 1. Also, the revenue curve here is much higher than that of network 2 (transit only network). The reason is that the mixed network has 2-hop originating and terminating traffic too which adds to the total revenue generated by network 2.

Thus the network service provider generates more revenue and is better off if he has a mixed network instead of transit only network. Will he still be better off if the end network before the mixed network increases its price? To see the effects let us see what happens if network 1 increases its flat price to 0.9.

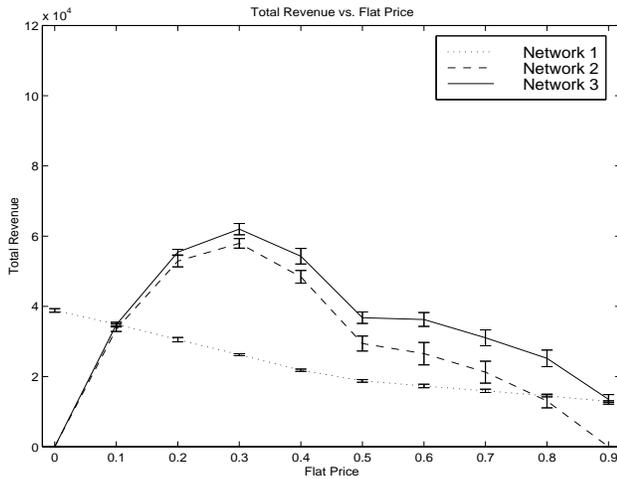


Figure 7: FFF: Total Revenue vs. Flat Prices of Networks 2 and 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.1 - Network 2 is Mixed

The level of flat price of network 1 is fixed at 0.9 in Figure 8. The revenue curves for networks 1 and 3 are almost the same as those of networks 1 and 3 when network 2 is transit only (Figure 6). Again, the main

difference lies between the revenue curves of network 2 in Figures 6 and 8. In Figure 8, the revenue curve of network 2 does not become zero at the point when the flat prices of networks 2 and 3 equal 0.1. After flat price of 0.1, network 2 can still transmit the traffic of 2-hop users of network 1. As flat prices of networks 2 and 3 reach 0.5, the revenue of network 2 becomes zero because the 2-hop users of network 2 can not send packets any more. When the flat prices of networks 2 and 3 are 0.5, the users of network 2 will face a price of 1.0. This price is higher than the packet values. So, no packets are sent. Also, the users of network 1 will not send any packets to networks 2 and 3 after flat prices of networks 2 and 3 become 0.1.

Though network 1 increased its flat price to 0.9, but network 2 (mixed network) can still generate revenue because of its terminating and originating traffic.

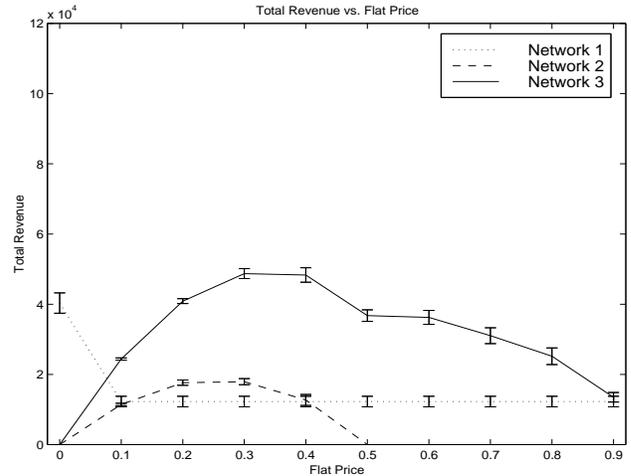


Figure 8: FFF: Total Revenue vs. Flat Prices of Networks 2 and 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.9 - Network 2 is Mixed

#### 4.1.2 Change Traffic - FFF

Do these results hold when traffic changes? To examine that question, we varied the traffic entering the network (assuming all networks use flat pricing); these are shown in Figures 9, 10, 11 and 12. Here, we show the total revenues of networks 1, 2, and 3 versus flat prices of networks 1, 2 and 3. The levels of flat prices of networks 1, 2, and 3 are varied between 0.0 and 0.9 in all cases.

Network 2 is a transit only network in Figures 9 and 10. The levels of  $\rho$  are 0.5 in Figure 9 for all the three networks. The revenue of networks 1 and 3 are almost the same because these two networks transmit almost the same number of packets. The revenue of network 2 becomes zero at flat price of 0.4 because the 3-hop users of network 1 cannot send their traffic through network 2 due to the high price that they

face. At this point, they face a flat price equal to 1.2 (0.4+0.4+0.4) which is higher than packet values.

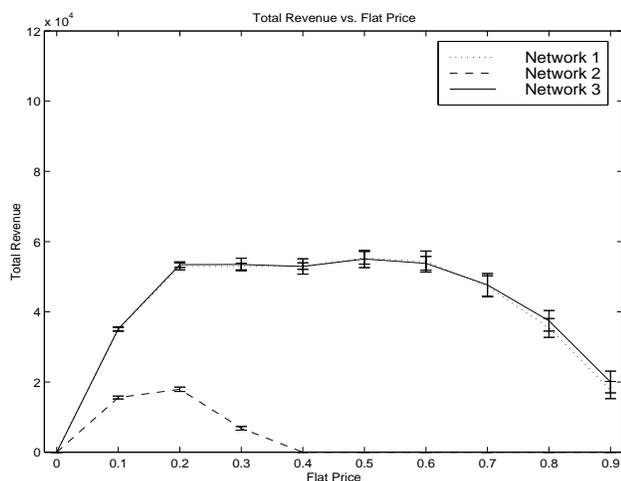


Figure 9: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 - All  $\rho=0.5$  - Network 2 is Transit Only

The levels of  $\rho$  are 0.9 in Figure 10 for all the three networks. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5 because at higher level of  $\rho$  more traffic enters the network. The revenues of network 1 and network 3 are also almost the same here. Also, the revenue of network 2 becomes zero at flat price of 0.4 due to the same reasons discussed in previous paragraph. Though the users want to send more traffic but they are constrained by price of 1.2.

These plots show that revenues increase as more traffic is added. The transit network service provider may be interested in generating even more revenue. He can do that by adding originating and terminating users to his network turning it into a mixed network. Now the revenue curves for mixed network will be discussed.

Network 2 is a mixed network in Figures 11 and 12. The levels of  $\rho$  are fixed at 0.5 in Figure 11 for all the three networks. The main difference is between the revenue curves of network 2 in Figures 9 and 11. In Figure 9, the revenue curve of network 2 (transit network) becomes zero at the point when the flat prices of the three networks are 0.4. In Figure 11, the revenue curve of network 2 (mixed network) becomes zero at the point when the flat prices of the three networks are 0.5. After flat price of 0.4, network 2 (mixed network) can still transmit the packets of the 2-hop users of network 1 and the 2-hop users of network 2. The revenue of network 2 becomes zero at flat price of 0.5 because now the 2-hop users of networks 1 and 2 also stop sending packets. Also, the revenue curve here is much higher than the revenue curve of network 2 (transit only network) due to the 2-hop originating

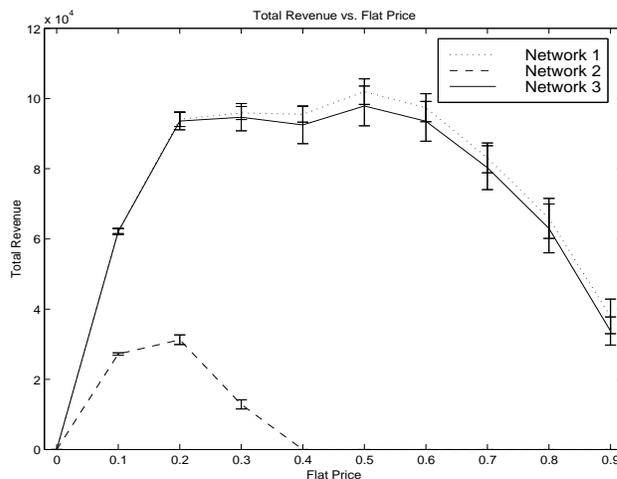


Figure 10: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 - All  $\rho=0.9$  - Network 2 is Transit Only

and terminating traffic.

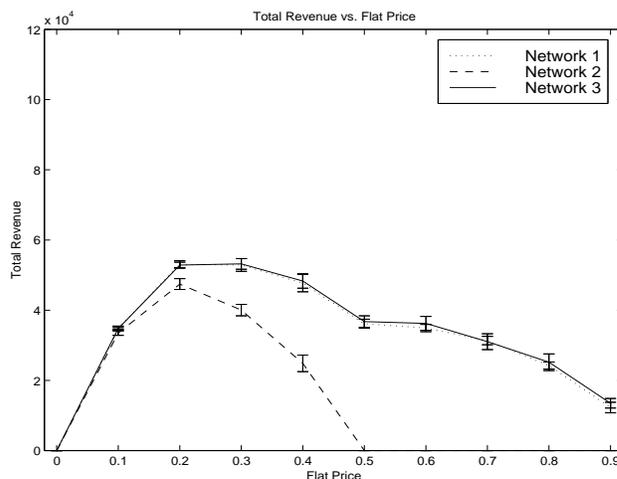


Figure 11: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 - All  $\rho=0.5$  - Network 2 is Mixed

The levels of  $\rho$  are 0.9 in Figure 12 for all the three networks. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5 because at higher levels of  $\rho$  more traffic enters the network. There is a difference between the shapes of revenue curves of networks 1 and 3 in Figure 10 and those of networks 1 and 3 in Figure 12. In Figure 10, the revenue curves of networks 1 and 3 do not go down much at the flat price of 0.5 whereas in Figure 12 they go down much at the flat price of 0.5. In Figure 10, networks 1 and 3 have only 1-hop and 3-hop users. At the flat price of 0.5, only the 3-hop users stop sending packets but the 1-hop users face the same price and keep sending

the same amount of traffic. In Figure 12, networks 1 and 3 have 1-hop, 2-hop, and 3-hop users. At the flat price of 0.5, the 3-hop as well as the 2-hop users stop sending packets. That is why the curves goes down more here.

Clearly, revenues increase as we add more traffic. This makes sense in the case of flat (per packet) pricing; to explore the consequences of capacity restriction, we conducted another set of similar experiments.

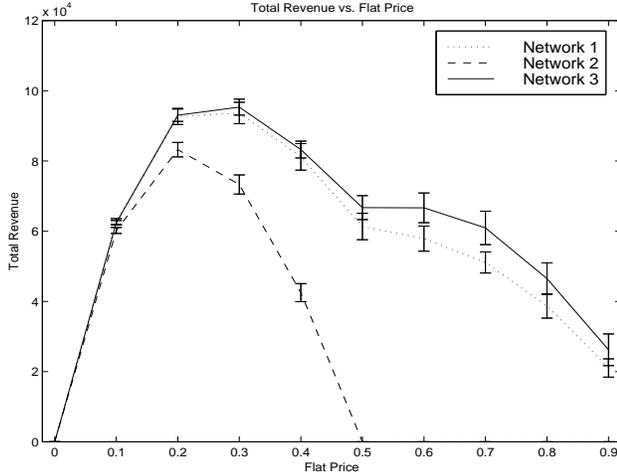


Figure 12: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 - All  $\rho_s=0.9$  - Network 2 is Mixed

#### 4.1.3 Change Capacity - FFF

The plots in Figures 13 and 14 show total revenues of networks 1, 2, and 3 versus flat prices of networks 1, 2, and 3. The levels of flat prices of networks 1, 2, and 3 are varied between 0.0 and 0.9 in all cases. The levels of capacity ( $C$ ) of networks 2 and 3 are medium in all cases.

Network 2 is a transit only network in Figure 13. The level of capacity of network 1 ( $C_1$ ) is low, which makes  $\rho$  of network 1 high. The revenue curves for networks 1, 2, and 3 have the same values and shapes for different levels of  $C_1$  that is why we have shown only one figure. Increasing and decreasing  $C_1$  does not affect the revenue curves in these figures. Though the capacities are changed, the amount of traffic that enters the network stays the same. Once packets enter the network, they are never dropped because of infinite queue sizes. These packets wait in the queues and are eventually transmitted. The same number of packets are transmitted by a network whether its capacity is low or high. The level of  $\rho$  of network changes by changing its capacity. The flat price of a network does not change when  $\rho$  of a network changes. That is why revenue of a network does not change when its capacity changes. This is true when a network is using

a flat pricing scheme. If a network is using congestion pricing scheme, its revenue will be affected by changing its capacity. This will be shown in sections 4.3.3 and 4.4.3. What is remarkable is that revenue flows are hardly affected. This may be a consequence of a relatively delay-insensitive user model.

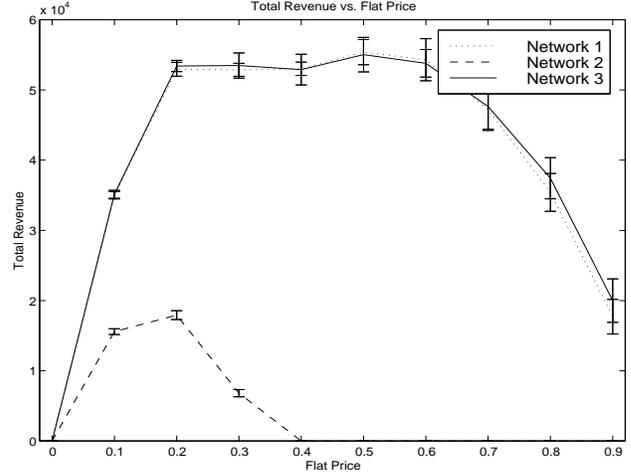


Figure 13: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 -  $C_1=Low$  - Network 2 is Transit Only

Network 2 is a mixed network in Figure 14.  $C_1$  is low which makes  $\rho$  of network 1 high. In this case too, the revenue curves for networks 1, 2, and 3 have the same values and shapes for different levels of  $C_1$  that is why we have shown only one figure. The revenue of network 2 (mixed network) in Figure 14 is much higher than that of network 2 (transit only network) in Figure 13 due to additional 2-hop originating and terminating users on the mixed network. It is not due to changing the capacity of network 1.

#### 4.2 Flat - Congestion - Flat (FCF)

In the previous section, we showed that a network service provider can change from transit only network to a mixed network to increase revenues. While this may or may not improve profitability due to potentially higher costs, it does reduce the provider's dependence on the business decisions of other networks.

An alternative to this is for the network service provider to switch to a congestion pricing scheme instead of flat pricing scheme. To examine the consequences of this, we conducted a set of experiments for our three network model with network 2 using a congestion pricing scheme and networks 1 and 3 using flat pricing schemes.

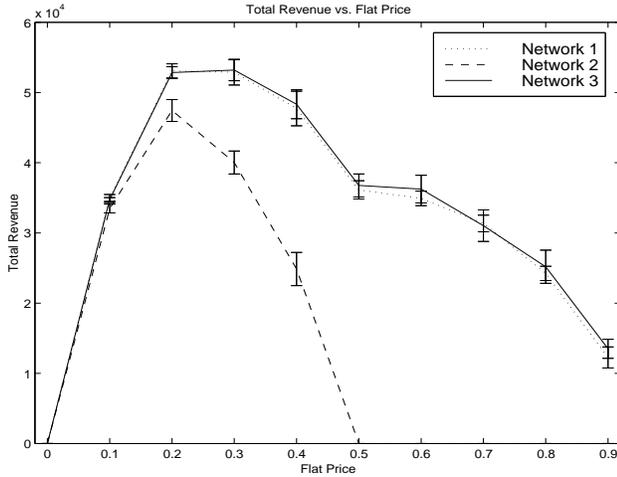


Figure 14: FFF: Total Revenue vs. Flat Prices of Networks 1, 2 and 3 -  $C1=Low$  - Network 2 is Mixed

#### 4.2.1 Change Flat Price - FCF

The plots in Figures 15, 16, 17, and 18 show total revenues of networks 1, 2, and 3 versus flat prices of networks 3. The levels of flat prices of networks 3 are varied between 0.0 and 0.9 in all cases. The levels of  $\rho$  are 0.5 in all cases.

Network 2 is a transit only network in Figures 15 and 16. The level of flat price of network 1 is fixed at 0.1 in Figure 15. The revenue curve for network 1 is maximum at  $3.2 \times 10^4$  and it stays almost the same and decreases slightly to  $2.0 \times 10^4$  as the levels of flat price of network 3 approach 0.9. In case of FCF, network 2 uses congestion pricing scheme. The revenue curve of network 2 in FCF is different from the revenue curves of network 2 in case of FFF. In FFF case, the revenue of network 2 is low at low flat prices, it increases as flat prices increase and then decreases with further increase of flat prices. In FCF case, revenue curve of network 2 is high at low flat prices and it decreases gradually until the flat prices reach 0.9. In congestion pricing scheme, the prices are not fixed as in case of flat pricing scheme. The prices of a network vary in response to the load on that network. When revenue of network 2 is highest ( $4.6 \times 10^4$ ), the flat price of network 3 is zero and the flat price of network 1 is 0.1. The 3-hop users of network 1 face a smaller price and they send many packets. These packets transit through network 2. The congestion price of network 2 goes high which results in high revenue for network 2. Revenues of network 2 decrease as the flat prices of network 3 increase. Now the 3-hop users of network 1 face higher prices and they send fewer packets through network 2. Revenue of network 3 is due to its own packets plus the packets of network 1 at lower flat prices of network 3. At higher flat price of network 3, network 3 collects revenue mostly of its own packets.

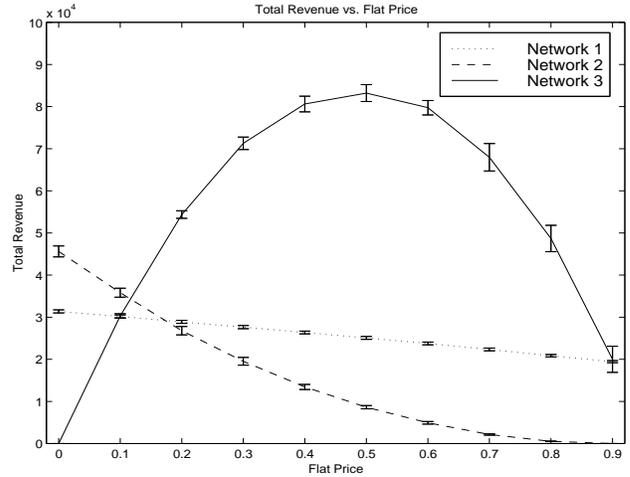


Figure 15: FCF: Total Revenue vs. Flat Price of Network 3 - All  $\rho=0.5$  - Flat Price of Network 1 = 0.1 - Network 2 is Transit Only

The level of flat price of network 1 is fixed at 0.5 in Figure 16. There is an increase in revenue of network 1 with a fixed flat price of 0.5 as compared to a flat price of 0.1 (Figure 15). The reason is that revenue is computed as the product of price and the number of packets transmitted. Though fewer packets are transmitted at flat price of 0.5 than at flat price of 0.1 but the higher price term (0.5) in the product increases the revenue. If the flat price is increased to 0.9, very few packets are transmitted and the price term is not effective in increasing the revenue as we observed in section 4.1.1. There is a decrease in revenue of network 2 as flat price of network 1 changes from 0.1 (Figure 15) to 0.5 (Figure 16). With the increase in flat price of network 1, the 3-hop users of network 1 send fewer packets through network 2 which reduces the congestion price of network 2. Lower congestion price results in lower revenues for network 2. Revenue of network 2 also decreases as the flat prices of network 3 increase. Revenue of network 3 decreases as flat price of network 1 changes from 0.1 to 0.5 because the 3-hop users of network 1 send fewer packets through networks 2 and 3.

Network 2 is a mixed network in Figures 17 and 18. The level of flat price is fixed at 0.1 in Figure 17. The revenue curves of networks 1 and 3 look almost the same in Figures 15 and 17. The main difference is between the revenue curves of network 2 in these two figures. In Figure 15, the revenue curve of network 2 (transit network) is much lower than that of network 2 (mixed network) in Figure 17. Though network 2 uses congestion pricing in both cases but network 2 has 2-hop originating and terminating traffic in case of mixed network. This traffic increases the level of congestion price of the mixed network and thus increases

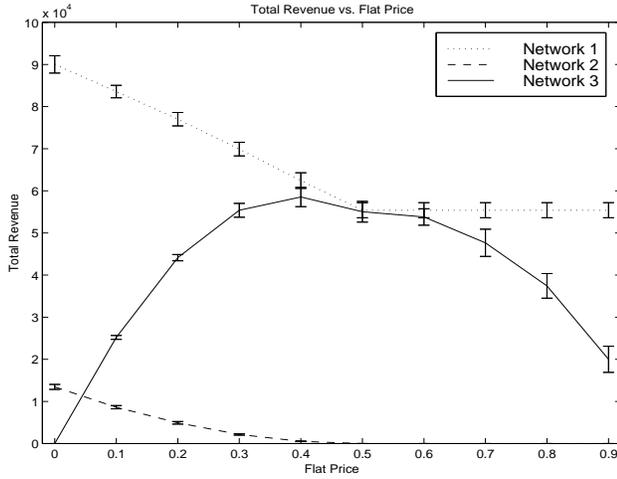


Figure 16: FCF: Total Revenue vs. Flat Price of Network 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.5 - Network 2 is Transit Only

its revenue.

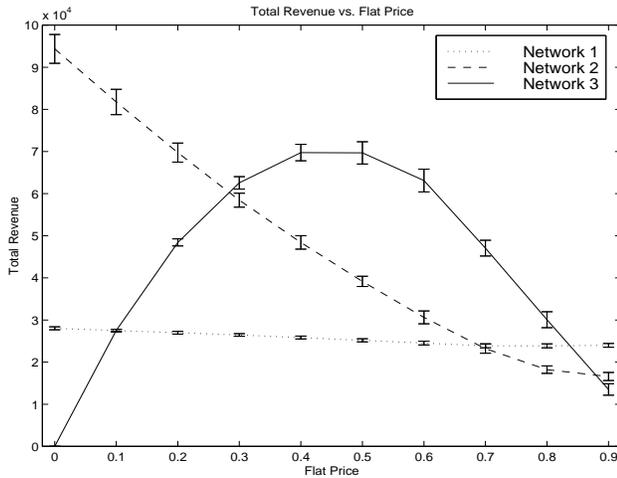


Figure 17: FCF: Total Revenue vs. Flat Price of Network 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.1 - Network 2 is Mixed

The level of flat price is fixed at 0.5 in Figure 18. There is an increase in revenue of network 1 with a flat price of 0.5 as compared to a fixed flat price of 0.1 (Figure 17) due to the same reasons as discussed for the behavior of revenue curves of network 1 in Figures 15 and 17. There is a decrease in revenue of network 2 as flat price of network 1 changes from 0.1 to 0.5. With the increase in flat price of network 1, the 2-hop and 3-hop users of network 1 send fewer packets through network 2 which results in lower congestion price and lower revenues for network 2. Revenue of network 3 also decreases as fixed flat price of network

1 changes from 0.1 to 0.5.

If networks 1, 2, and 3 are owned by three independent Internet Service Providers, there will be a conflict of interests between networks 1 and 2. Network 1 generates higher revenue at a flat price of 0.5 and lower revenue at a flat price of 0.1 whereas network 2 generates lower revenue at a flat price of 0.5 and higher revenue at a flat price of 0.1. This would seem to suggest that incentives exist for Networks 1 and 2 to enter into a contractual arrangement outside of the pricing schemes we study here.

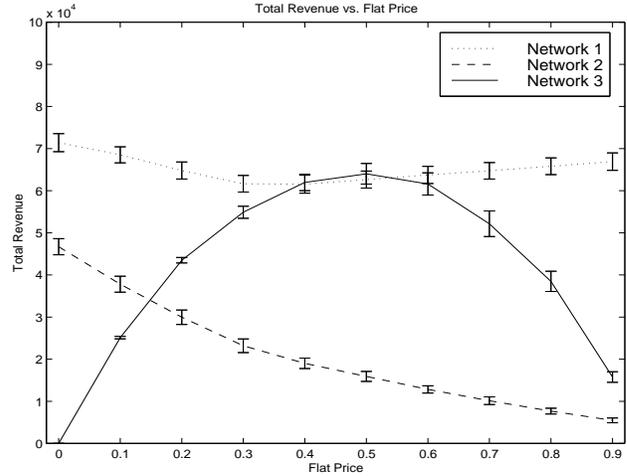


Figure 18: FCF: Total Revenue vs. Flat Price of Network 3 - All  $\rho_s=0.5$  - Flat Price of Network 1 = 0.5 - Network 2 is Mixed

#### 4.2.2 Change Traffic - FCF

The plots in Figures 19, 20, 21, and 22 show total revenues of networks 1, 2, and 3 versus flat prices of networks 1 and 3. The levels of flat prices of networks 1 and 3 are varied between 0.0 and 0.9 in all cases.

Network 2 is a transit only network in Figures 19 and 20. The levels of  $\rho$  are 0.5 in Figure 19 for all the three networks. The revenues of network 1 and network 3 are almost the same because these two networks transmit almost the same number of packets. The revenue curve of network 2 is high at low flat prices and it decreases gradually until the flat prices reach 0.9. The congestion price of network 2 stays the same for the same levels of  $\rho$ . As the flat prices of networks 1 and 3 increase, the 3-hop users of network 1 start to send fewer packets through network 2 and network 2 starts collecting decreasing revenues.

The levels of  $\rho$  are 0.9 in Figure 20 for all the three networks. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5 because at higher levels of  $\rho$  more traffic enters the network. The revenue curve of network 2 is at maximum when the flat prices of network 1 and 3 are zero. Since the flat price

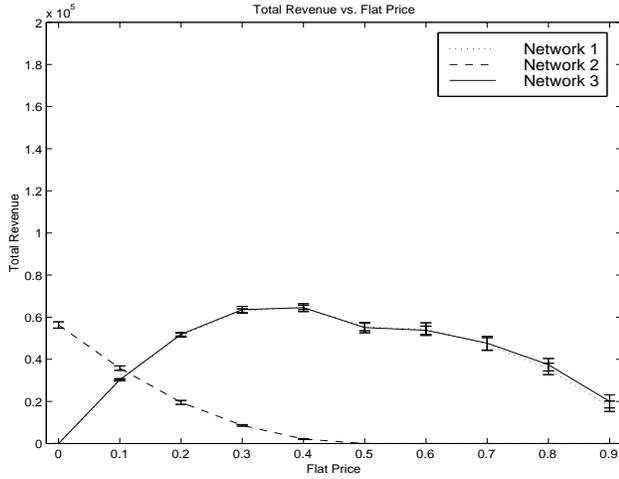


Figure 19: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 - All  $\rho=0.5$  - Network 2 is Transit Only

of network 1 is zero, the 3-hop users of network 1 are sending many packets through network 2. The congestion price of network 2 goes high due to packets of network 1. Network 2 collects higher revenue due to higher congestion price.

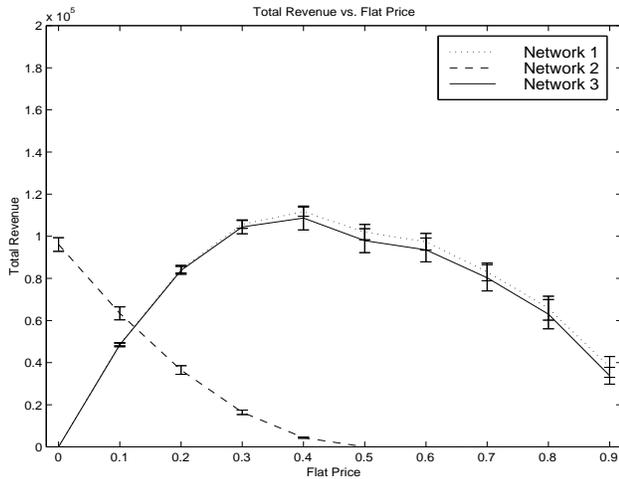


Figure 20: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 - All  $\rho=0.9$  - Network 2 is Transit Only

Network 2 is a mixed network in Figures 21 and 22. The levels of  $\rho$  are 0.5 in Figure 21 for all the three networks. The revenue curve of network 2 (mixed network) is higher in Figure 21 than that of network 2 (transit only network) in Figure 19.

The levels of  $\rho$  are 0.9 in Figure 22 for all the three networks. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5. The revenue

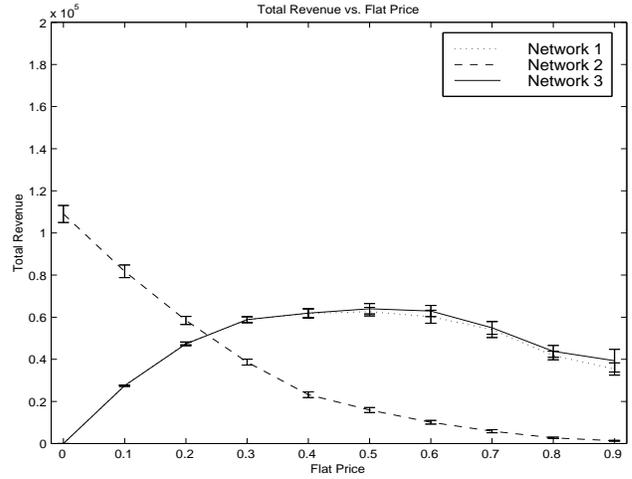


Figure 21: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 - All  $\rho=0.5$  - Network 2 is Mixed

curve of network 2 (mixed network) is higher in Figure 22 than that of network 2 (transit only network) in Figure 20. The revenue curve of network 2 is at maximum when the flat prices of network 1 and 3 are zero. Since the flat price of network 1 is zero, the 2-hop and 3-hop users of network 1 are sending many packets through network 2. The congestion price of network 2 goes high due to packets of network 1 and its own packets. Network 2 collects higher revenue due to higher congestion price.

It is obvious that the network service provider operating network 2 makes very high revenue by adopting a congestion pricing scheme and extending his network to a mixed network.

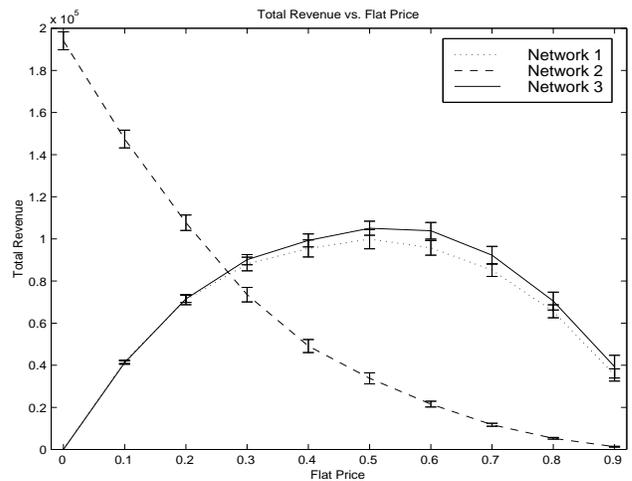


Figure 22: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 - All  $\rho=0.9$  - Network 2 is Mixed

### 4.2.3 Change Capacity - FCF

The plots in Figures 23 and 24 show total revenues of networks 1, 2, and 3 versus flat prices of network 3. The levels of flat prices of network 3 are varied between 0.0 and 0.9 in all cases. The levels of capacity of networks 2 and 3 are medium in all cases.

Network 2 is a transit only network in Figure 23. In this Figure, the level of  $C1$  is low which makes  $\rho$  of network 1 high. The revenue curves for networks 1, 2, and 3 have the same values and shapes for different levels of  $C1$  that is why we have shown only one figure. Increasing and decreasing  $C1$  does not affect the revenue curves in these figures due to the same reasons as described in section 4.1.3.

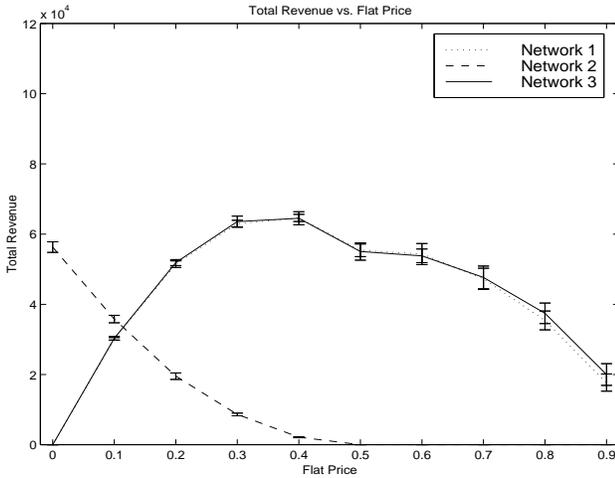


Figure 23: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 -  $C1=Low$  - Network 2 is Transit Only

Network 2 is a mixed network in Figure 24. In this Figure, the level of  $C1$  is low which makes  $\rho$  of network 1 high. The revenue curves for networks 1, 2, and 3 have the same values and shapes for different levels of  $C1$  that is why we have shown only one figure. The revenue curve of network 2 (mixed network) in Figure 24 higher than that of network 2 (transit only network) in Figure 23. The reason is that more traffic enters network 2 (mixed network) due to the 2-hop originating and terminating traffic of network 2.

The end network service providers may want to implement congestion pricing schemes too after seeing the benefits of congestion pricing scheme implemented by the middle network service provider. In the next section, we discuss the effects produced due to congestion pricing schemes implemented by networks 1 and 2.

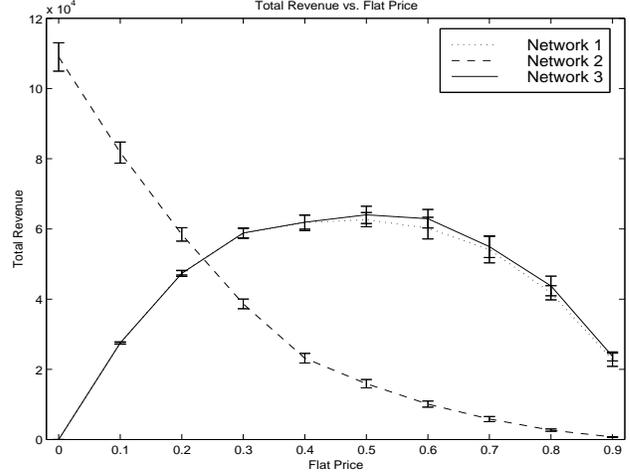


Figure 24: FCF: Total Revenue vs. Flat Prices of Networks 1 and 3 -  $C1=Low$  - Network 2 is Mixed

### 4.3 Congestion - Flat - Congestion (CFC)

In this case the first network uses congestion pricing, the second network uses flat pricing, and the third network uses congestion pricing scheme.

#### 4.3.1 Change Flat Price - CFC

Here the levels of flat prices of network 2 can be varied between 0.0 and 0.9. The effects produced due to changing the flat prices of network 2 are discussed in the next section 4.3.2 along with the effects produced due to changing the traffic of networks.

#### 4.3.2 Change Traffic - CFC

The plots in Figures 25, 26, 27, and 28 show total revenues of networks 1, 2, and 3 versus flat prices of network 2. The levels of flat price of network 2 are varied between 0.0 and 0.9 in all cases.

Network 2 is a transit only network in Figures 25 and 26. The levels of  $\rho$  are 0.5 in Figure 25 for all the three networks. The revenue curves for networks 1 and 3 look the same because networks 1 and 3 use congestion pricing schemes and both networks have same levels of  $\rho$ . The revenue curve of network 2 becomes zero as the flat price of network 2 approaches 0.6. After flat price of 0.6, the revenue curves of networks 1 and 3 become constant and remain constant until the flat price of 0.9. The reason is that after this price, the 3-hop users of network 1 stop sending packets to network 2 and 3. Now the revenues of networks 1 and 3 are only because of their 1-hop users. With a higher level of  $\rho$ , the congestion prices of networks 1 and 3 are higher. The price faced by 3-hop users of network 1 goes high because of high congestion prices

of networks 1 and 3. These users reduce sending packets through network 2 when the price goes higher than 1.0. Revenues of networks 1 and 3 are high due to their high congestion prices.

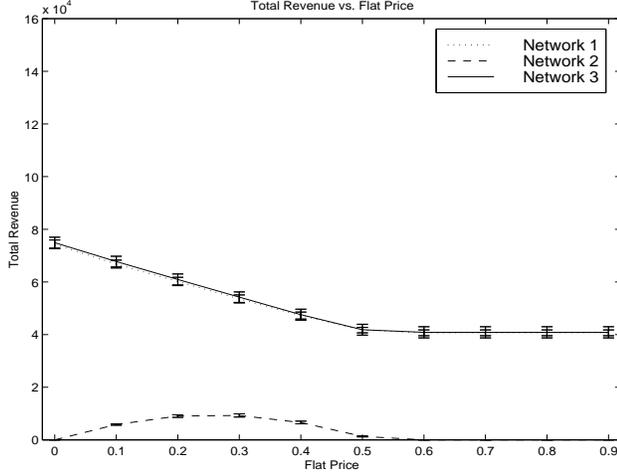


Figure 25: CFC: Total Revenue vs. Flat Price of network 2 - All  $\rho=0.5$  - Network 2 is Transit Only

The levels of  $\rho$  are 0.9 in Figure 26 for all the three networks. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5 because at higher levels of  $\rho$  more traffic enters the network. The revenue curve of network 2 becomes zero as the flat price of network 2 approaches 0.3. It goes to zero earlier than it does when  $\rho$  is 0.5 (Figure 25). Now the level of  $\rho$  is 0.9 which makes the congestion prices of networks 1 and 3 higher than those when  $\rho$  is 0.5. The 3-hop users of network 1 send fewer packets through network 2 due to higher congestion prices of networks 1 and 3. That is why the revenue curve of network 2 is smaller now and also becomes zero at a lower flat price (0.3). The revenues of networks 1 and 3 decrease as the flat price of network 2 approaches 0.3. After this point the revenues of networks 1 and 3 are constant because the congestion prices of both networks are also almost the same for same value of  $\rho$  and both networks are transmitting their own packets only. Networks 1 and 3 collect more revenues than they do when  $\rho$  is 0.5 due to higher congestion prices.

Network 2 is a mixed network in Figures 27 and 28. The levels of  $\rho$  are 0.5 in Figure 27 for all the three networks. The revenue curve of network 2 (mixed network) is much higher than that of network 2 (transit only network) in Figure 25. The revenue curve of network 2 (mixed network) does not become zero at flat price of 0.6; it becomes zero at flat price of 0.9. This is an advantage of a mixed network over a transit only network.

The levels of  $\rho$  are 0.9 in Figure 28 for all the three networks. The revenues of all the three networks are

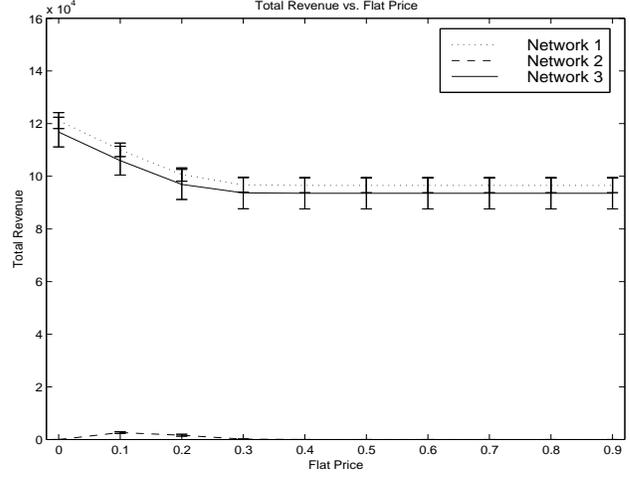


Figure 26: CFC: Total Revenue vs. Flat Price of network 2 - All  $\rho=0.9$  - Network 2 is Transit Only

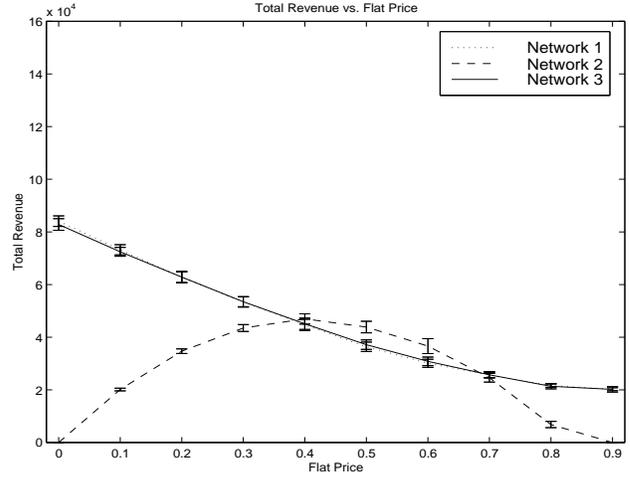


Figure 27: CFC: Total Revenue vs. Flat Price of network 2 - All  $\rho=0.5$  - Network 2 is Mixed

higher than their revenues at  $\rho$  of 0.5 (Figure 27) because at higher level of  $\rho$  more traffic enters the network. Here the revenue curve of network 2 (mixed network) is also much higher than that of network 2 (transit only network) in Figure 26. The revenue curves of networks 1 and 3 are at maximum when the flat price of network 2 is zero. Since the flat price of network 2 is zero, the 2-hop and 3-hop users of network 1 are sending many packets through network 2. The congestion prices of networks 1 and 3 go high due to large number of packets transmitted by these networks. Networks 1 and 3 collect high revenue due to high congestion prices. The revenue curves of network 1 and 3 decrease as flat price of network 2 increases because now the 2-hop and 3-hop users of network 1

start sending fewer packets through network 2.

The results show that the decision made by the end network service providers to implement congestion pricing schemes is very useful for them if the traffic on their networks increases. At the same time, this decision is very detrimental to the interests of the middle network service provider who is using flat pricing scheme. The high volume of traffic increases the congestion prices of end network service providers and thus they generate more revenue. Due to these high congestion prices less traffic goes through the middle network.

These end network service providers now have the incentive to change the capacity of their networks in order to affect the congestion prices and make profits. Now we discuss the consequences when one of the end network service providers changes the capacity of his network.

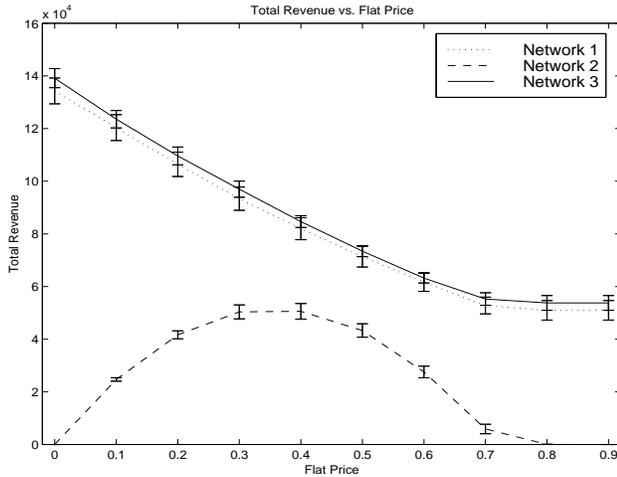


Figure 28: CFC: Total Revenue vs. Flat Price of network 2 - All  $\rho$ s=0.9 - Network 2 is Mixed

### 4.3.3 Change Capacity - CFC

The plots in Figures 29, 30, 31, and 32 show total revenues of networks 1, 2, and 3 versus flat prices of network 2. The levels of flat prices of network 2 are varied between 0.0 and 0.9 in all cases. The levels of capacity of networks 2 and 3 are medium in all cases.

Network 2 is a transit only network in Figures 29 and 30. The level of  $C1$  is high in Figure 29. The main point to note here is that the revenue curves for network 1 have different values for different levels of  $C1$  in these figures as opposed to revenue curves of network 1 in cases of FFF (section 4.1.3) and FCF (section 4.2.3). The revenue curves of network 2 and 3 are also affected by changing  $C1$ . High level of  $C1$  makes  $\rho$  of network 1 low. Congestion price of network 1 is low when its  $\rho$  is low. Network 1 collects low

revenue due to its low congestion price. The revenue curve of network 3 is higher than that of network 1. The reason is that the capacity of network 3 is lower than that of network 1 resulting in higher  $\rho$  for network 3. Congestion price of network 3 is higher when its  $\rho$  is higher. Network 3 collects higher revenue due to its higher congestion price.

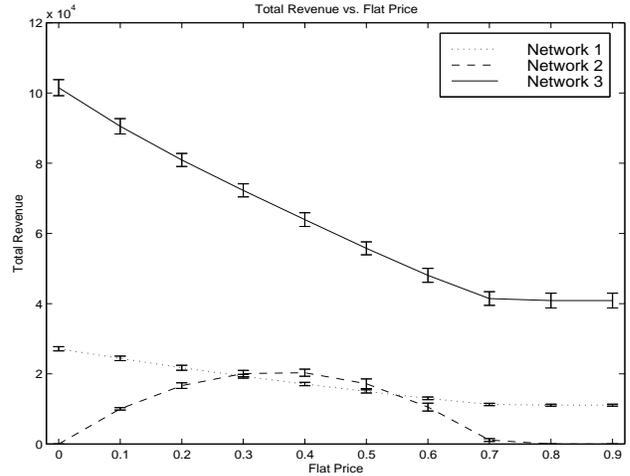


Figure 29: CFC: Total Revenue vs. Flat Price of Network 2 -  $C1$ =High - Network 2 is Transit Only

The level of  $C1$  is low in Figure 30. The revenue curve for network 1 is higher than its revenue curve when  $C1$  is high (Figure 29). Low level of  $C1$  now makes  $\rho$  of network 1 high. Congestion price of network 1 is higher when its  $\rho$  is higher. Network 1 collects higher revenue due to its higher congestion price. Now the behavior of revenue curve of network 1 in cases of FFF (section 4.1.3) and FCF (section 4.2.3). In cases of FFF and FCF, network 1 uses flat pricing scheme. In case of flat pricing scheme, increasing and decreasing  $C1$  does not affect the revenue curves. In flat pricing scheme, the price does not change in response to  $\rho$ . That is why revenue does not change. If capacity of a network is increased or decreased, its  $\rho$  decreases or increases for the same number of packets entering the network. In our model, packets are never dropped once they enter the network. So, the same number of packets are transmitted for both high and low levels of capacity. In case of CFC, network 1 uses congestion pricing scheme and congestion price is affected by the values of capacity and  $\rho$ . If capacity of a network changes, its  $\rho$  changes. If  $\rho$  of a network changes, its congestion price changes because that is how congestion prices are computed. If the congestion price of a network changes, its revenue changes because revenue is simply the product of price of a network and the packets transmitted by that network. The revenue curves for networks 2 and 3 are lower than their corre-

sponding revenue curves when  $C1$  is high (Figure 29). Now the 3-hop users of network 1 send fewer packets through network 2 due to higher congestion price of network 1. The revenue curve of network 3 is lower than that of network 1. The reason is that the capacity of network 3 is lower than that of network 1 resulting in lower  $\rho$  for network 3. Congestion price of network 3 is lower when its  $\rho$  is lower. Network 3 collects lower revenue due to its lower congestion price.

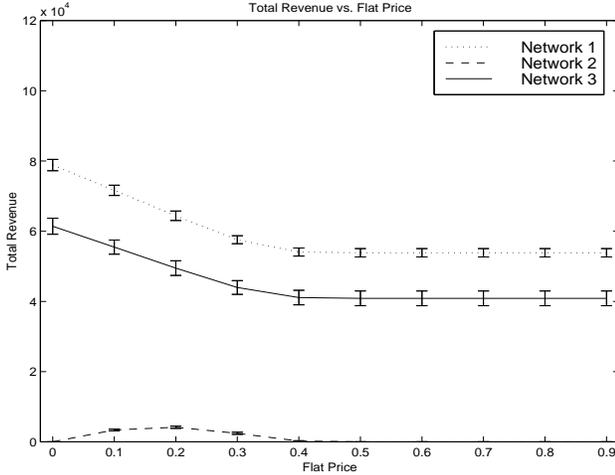


Figure 30: CFC: Total Revenue vs. Flat Price of Network 2 -  $C1=Low$  - Network 2 is Transit Only

Network 2 is a mixed network in Figures 31 and 32. The level of  $C1$  is high in Figure 31. The revenue curve for network 2 (mixed network) in Figure 31 is higher than that of network 2 (transit only network) in Figure 29. Congestion price of network 1 is low due to its low  $\rho$  resulting in low revenue for network 1. Now the 2-hop and 3-hop users of network 1 send more traffic through network 2 because of low congestion price of network 1. Also, the 2-hop users of network 1 send packets through network 2. This results in high revenue for network 2 (mixed network).

The level of  $C1$  is low in Figure 32. The capacity of network 1 is low now which makes  $\rho$  of network 1 high. Here, the revenue curve for network 2 (mixed network) in Figure 32 is also higher than that of network 2 (transit only network) in Figure 30 due to the same reasons as described in previous paragraph.

A network service provider may have the incentive to reduce the capacity of its network in order to create congestion artificially. The users face higher congestion prices due to congestion but the network service provider generates more revenue. This problem can be handled by competition and regulatory policies.

All the three network service providers want to use congestion pricing schemes. In the next section, we will discuss the implications of changing traffic and capacity when all the three networks use congestion

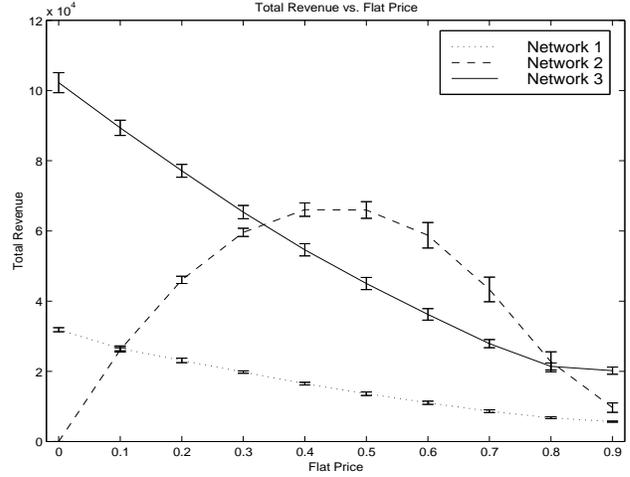


Figure 31: CFC: Total Revenue vs. Flat Price of Network 2 -  $C1=High$  - Network 2 is Mixed

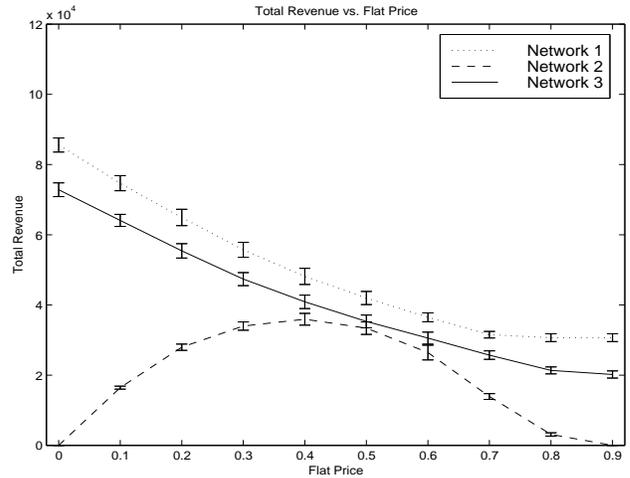


Figure 32: CFC: Total Revenue vs. Flat Price of Network 2 -  $C1=Low$  - Network 2 is Mixed

pricing schemes.

## 4.4 Congestion - Congestion - Congestion (CCC)

In this case all the three networks use congestion pricing schemes. Since none of the networks uses flat pricing schemes, Tables are used instead of Figures in this section because revenues cannot be plotted versus flat prices.

### 4.4.1 Change Flat Price - CCC

Changing flat price does not apply in this case because all the three networks use congestion pricing schemes.

#### 4.4.2 Change Traffic - CCC

The values in Tables 1 and 2 show total revenues of networks 1, 2, and 3 for different levels of  $\rho$ . Network 2 is a transit only network in Table 1. The levels of  $\rho$  are 0.5 in the first rows for each of the networks in Table 1. The revenues of network 1 and network 3 are almost the same because these two networks transmit almost the same number of packets.

The levels of  $\rho$  are 0.9 in second rows for each of the networks in Table 1. With higher levels of  $\rho$  more traffic enters a network. The congestion price of a network increases with the increase in traffic. Higher congestion price generates higher revenue. This is true for networks 1 and 3 because these networks have 1-hop users which do not face higher congestion prices due to higher levels of  $\rho$ . In case of network 2, the revenue is lower at higher levels of  $\rho$ . The reason is that now the 3-hop users of network 1 stop sending packets through network 2 due to higher congestion prices of networks 1, 2, and 3. The reduction in number of packets sent through network 2 reduces the revenue for network 2.

	<i>Total Revenue</i>
	Network 1: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	64097.7 (62564.2, 65631.1)
$\rho_1=\rho_2=\rho_3=0.9$	112501.4 (109766.0, 115236.7)
	Network 2: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	7548.3 (7136.4, 7960.1)
$\rho_1=\rho_2=\rho_3=0.9$	2658.3 (2340.2, 2976.4)
	Network 3: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	65030.9 (62782.8, 67279.0)
$\rho_1=\rho_2=\rho_3=0.9$	108249.4 (102339.0, 114159.8)

Table 1: CCC: Revenues of Networks for different  $\rho$ s - Network 2 is Transit Only

Network 2 is a mixed network in Table 2. The levels of  $\rho$  are 0.5 in first rows for each of the networks in Table 2. The revenues of networks 1 and 3 in Table 2 are lower than those of networks 1 and 3 in Table 1 for the same levels of  $\rho$ . The revenue of network 2 (mixed network) in Table 2 is higher than that of network 2 (transit only network) in Table 1. Now the congestion price of network 2 is higher due to its 2-hop originating and terminating traffic. This results in higher revenue for network 2. At the same time, the 2-hop and 3-hop users of network 1 send less traffic through network 2 due to its high price. That is why the revenues for network 1 and 3 are lower.

The levels of  $\rho$  are 0.9 in second rows under each network section in Table 2. The revenues of all the three networks are higher than their revenues at  $\rho$  of 0.5 because at higher level of  $\rho$  more traffic enters the network. Higher level of  $\rho$  results in higher congestion prices which in turn result in higher revenue. The behavior of network 2 (mixed network) in Table 2 is just

opposite to that of network 2 (transit only network) in Table 1. In case of network 2 (mixed network), the revenue is higher at higher levels of  $\rho$ . The reason is that now network 2 has 2-hop originating and terminating users too. Even though congestion prices are still high but these 2-hop users face lower congestion prices as compared to congestion prices faced by the 3-hop users of network 1 in Table 1.

A transit only network service provider is always at disadvantage if the end network service provider uses congestion pricing and the traffic increases. In such a situation, the congestion prices of all the networks increase. The end network service provider generates higher revenue due to higher congestion prices. The transit only network service provider does not receive much traffic due to high congestion prices of end network service providers.

	<i>Total Revenue</i>
	Network 1: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	59263.2 (57420.1, 61106.2)
$\rho_1=\rho_2=\rho_3=0.9$	97462.9 (92646.9, 102278.9)
	Network 2: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	38825.7 (37504.9, 40146.6)
$\rho_1=\rho_2=\rho_3=0.9$	49069.7 (46400.5, 51739.0)
	Network 3: Mean (C.I)
$\rho_1=\rho_2=\rho_3=0.5$	59466.1 (57538.4, 61393.8)
$\rho_1=\rho_2=\rho_3=0.9$	101680.2 (98264.4, 105096.0)

Table 2: CCC: Revenues of Networks for different  $\rho$ s - Network 2 is Mixed

#### 4.4.3 Change Capacity - CCC

The values in Tables 3 and 4 show total revenues of networks 1, 2, and 3 for different levels of capacity  $C$ . The levels of capacity of networks 2 and 3 are medium in all cases.

Network 2 is a transit only network in Table 3. The level of  $C_1$  is high in the first row for each of the networks in Table 3. High level of  $C_1$  makes  $\rho$  of network 1 low. Congestion price of network 1 is low when its  $\rho$  is low. Network 1 collects low revenue due to its low congestion price. The revenue of network 2 is lower than those of networks 1 and 3. When the congestion prices of networks 2 and 3 are high, the 1-hop users of network 1 send fewer packets through network 2. The revenue of network 3 is higher than that of network 1. The reason is that the capacity of network 3 is lower than that of network 1 resulting in higher  $\rho$  for network 3. Congestion price of network 3 is higher when its  $\rho$  is higher. Network 3 collects higher revenue due to its higher congestion price.

The level of  $C_1$  is low in the second row for each of the networks in Table 3. The revenue for network 1

is higher than its revenue when  $C1$  is high. Now the lower level of  $C1$  makes  $\rho$  of network 1 high. Congestion price of network 1 is higher when its  $\rho$  is higher. Network 1 collects higher revenue due to its higher congestion price. The revenue for network 2 is lower than its revenue when  $C1$  is high. This is due to the higher congestion price of network 1. Now the 3-hop users of network 1 send fewer packets through network 2 due to higher price. The revenue of network 3 is lower than that of network 1. The reason is that the capacity of network 3 is lower than that of network 1 resulting in lower  $\rho$  for network 3. Congestion price of network 3 is lower when its  $\rho$  is lower. Network 3 collects lower revenue due to its lower congestion price.

	<i>Total Revenue</i>
	Network 1: Mean (C.I)
$C1=H,C2=C3=M$	21095.9 (20498.7, 21693.1)
$C1=L,C2=C3=M$	72585.9 (71161.3, 74010.5)
	Network 2: Mean (C.I)
$C1=H,C2=C3=M$	17975.5 (16937.5, 19013.4)
$C1=L,C2=C3=M$	3191.5 (2897.1, 3485.8)
	Network 3: Mean (C.I)
$C1=H,C2=C3=M$	78721.4 (76654.3, 80788.4)
$C1=L,C2=C3=M$	56154.4 ( 54021.6, 58287.1)

Table 3: CCC: Revenues of Networks for different  $Cs$  - Network 2 is Transit Only

Network 2 is a mixed network in Table 4. The level of  $C1$  is high in the first row for each of the networks in Table 4. The revenue of network 2 (mixed network) in Table 4 is higher than that of network 2 (transit only network) in Table 3. Now network 2 (mixed network) has 2-hop originating and terminating users. The revenue of network 3 is higher than that of network 1 due to the same reasons discussed for revenues of network 1 and 3 in Table 3.

The level of  $C1$  is low in the second row for each of the networks in Table 4. The revenue for network 1 is higher than its revenue when  $C1$  is high. Network 1 collects higher revenue due to its higher congestion price. The revenue for network 2 is lower than its revenue when  $C1$  is high. Now the 2-hop and 3-hop users of network 1 send fewer packets through network 2 due to higher congestion price of network 1. The revenue of network 3 is lower than that of network 1. Network 3 collects lower revenue due to its lower congestion price.

## 5 Discussion and Conclusion

In this paper, we study the behavior of networks in terms of their revenue flows or settlements under multiple service providers. Each of these networks can

	<i>Total Revenue</i>
	Network 1: Mean (C.I)
$C1=H,C2=C3=M$	19847.5 (19537.5, 20157.6)
$C1=L,C2=C3=M$	65075.2 (62989.8, 67160.6)
	Network 2: Mean (C.I)
$C1=H,C2=C3=M$	60408.4 (58806.8, 62010.0)
$C1=L,C2=C3=M$	27996.8 (26796.5, 29197.1)
	Network 3: Mean (C.I)
$C1=H,C2=C3=M$	65331.2 (63555.1, 67107.4)
$C1=L,C2=C3=M$	55785.7 (53894.1, 55785.7)

Table 4: CCC: Revenues of Networks for different  $Cs$  - Network 2 is Mixed

change its pricing strategies, flat price levels, its traffic, and capacity. The effects of these changes on other networks are shown.

If a network service provider uses flat pricing scheme, he has the incentive to change the levels of flat prices in order to maximize his profits by generating more revenue. A network service provider does not like a flat price of zero because the network does not generate any revenue at zero flat price. The revenue of a network increases with the increase of its flat price until the flat price reaches some optimal level and after this level the revenue decreases with the increase of flat price. The network service provider would like to keep the flat price at the level where the revenue is maximum. Changing the levels of flat price of a network also affects the traffic and revenues of other networks in the chain. These other networks can be transit only networks, mixed networks, and networks which are  $n$  hops away. If flat price of a network is zero or low, this network may send more packets to other networks in the chain depending on the price of other networks. The low flat price of a network may not collect high revenue for itself but it may help other networks to collect high revenue. If the other networks on the chain use congestion pricing schemes or the optimal levels of flat pricing schemes, these network will collect more revenue due to high traffic coming from the first network because of its zero or low flat price. If the other network on the chain is transit only network, it is better off if the networks before it on the chain use lower levels of flat prices. Transit only network collects high revenue if other networks transmit more packets through it. On the other hand, if the flat price of a network is high it will not send packets to other networks. It will not help other networks to collect revenue. If the other network on the chain is a mixed network, it can still generate revenue due to its originating and terminating traffic. A mixed network is still affected by the high flat prices of the end network but it can generate revenue due to its own traffic too. Increasing the traffic of a network increases the

revenue for that network. If this traffic goes through other networks, it also increases the revenue for other networks. Changing the capacity of a network does not affect its revenue for a fixed amount of its traffic. In our model, queues have infinite sizes so packets are not dropped once they enter the network. The change of capacity changes the  $\rho$  of a network. Flat price of a network does not change when its  $\rho$  varies. That is why revenue does not change with the change of capacity in case of flat pricing scheme.

A network service provider may have the incentive to use congestion pricing instead of flat pricing in order to generate more revenue. If a network uses congestion pricing scheme, it always collects nonzero revenue as long as it transmits some traffic. Increasing the traffic of a network increases the congestion price of that network. Revenue of a network increases with the increase of its congestion price. A network service provider would be happy with more traffic but this has some implications for this network as well as for other networks on the chain. If a network sends more traffic, its congestion price will go higher and its users will send fewer packets resulting in decreasing revenue. If the other network on the chain is a transit only network, its congestion price will be higher due to higher traffic. Now the first network will send fewer packets through it and the revenue of transit only network will be lower. If the other network is a mixed network, its congestion price can still go high and the first network will send less traffic through it. But a mixed network has its own 2-hop users who face lower prices. Thus a mixed network can generate more revenue than transit only network. Changing the capacity of a network affects its revenue for a fixed amount of its traffic. Increasing the capacity of network reduces its  $\rho$  for a fixed amount of its traffic. Congestion price of a network increases or decreases with the increase or decrease of its  $\rho$ . Revenue of a network increases or decreases with the increase or decrease of its congestion price. Increasing capacity of a network collects low revenue for that network. On the other hand, increasing capacity of a network may help other networks on the chain to collect more revenues. The reason is that if congestion price of a network is low because of its high capacity, its users may face low price and they send more packets through other networks and the revenues of these networks increase.

## 6 Future Work

Our future work will address pricing issues in real time networks supporting multiple service classes and multimedia applications with Quality of Service (QoS) requirements. The interactions and settlements among these kinds of networks will also be investigated.

It is expected that the outcome of this research will inform public policy development process and the

standards development process to aid in the implementation of effective standards and efficient public policy in the area of computer networks.

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