Duopoly Interaction and Expected Price for Local Access

Running Heading: Duopoly Interaction and Expected Price for Local Access

Judith A. Molka-Danielsen
Department of Informatics
Molde College
P.O. Box 308, N-6401 Molde, Norway
Tel. +47 71 21 42 39 Fax + 47 71 21 42 00
email: molka@hiMolde.no

Martin B.H. Weiss
Telecommunications Program
University of Pittsburgh
135 N. Bellefield Ave., Pittsburgh, PA 15260
USA
Tel. 1 412 624 9405
email: mbw@pitt.edu

1 Introduction

One of the important goals of the Telecommunications Act of 1996 is competition in the local exchange. As this competition is introduced, traditional subsidies, explicit and implict, will most likely disappear. For example, geographic averaging created an arbitrage opportunity in urban areas, one that has been taken advantage of by entrants since the early 1990s. Competition brings pricing flexibility, so that one could expect implicit subsidies, like geographic averaging, to disappear. For policymakers, an important question is "What happens to universal service as pricing flexibility is introduced?" A large impact on universal service would imply the need for a system of explicit subsidies to meet public policy goals.

In this paper, we consider what impact cost-based pricing (without geographic averaging) in the absence of other subsidies might have on universal service. To do this, we estimate what prices consumers might pay (and the associated penetration rates) for local access if two firms vie for their business assuming no universal service subsidy. Given the parameters of the study, the expected penetration rates for households within the study areas would be substantially lower than the current U.S. average household penetration rate, which suggests that liberalization and deregulation of local access pricing requires a corresponding need for access price subsidies if current penetration rates are to be maintained.

Put formally, in this paper we conduct an evaluation of expected price for access to local telephone service when there is duopoly competition.[i] Using Green and Porter's model (Green and Porter, 1984), we compute the quantities the two firms will choose to produce to maximize their present and future expected value given their strategies. We apply the resultant prices to a model for access demand that was developed by Perl to estimate penetration rates (Perl, 1983; Perl, 1984).

Many technologies exist to implement facilities-based networks that might compete with incumbents, such as CATV companies, fixed wireless, *etc*, which suggests that the cost base of the incumbent and that of an entrant would be different from the incumbent. We chose a CATV provider, for which Reed developed a cost model (Reed, 1993). For the incumbent's cost, we use our own study of investment costs to compute the total capital costs of access and, using U.S. Census data, compute an average network cost per subscriber for specific 14 service areas.

In addition to high network investment costs for new entrants, new entrants must match the large service portfolios of the incumbent LECs, which will increase the entry costs for local access. Because of these high costs, we assume the number of facilities-based competitors in most geographic markets will be few, so we consider the duopoly assumption to be reasonable for many markets.

We then simulate the behavior of two competing providers using an interactive game model where parameters were selected to match a current U.S. local access telephone market. We compute the expected price for access for a household and compute the future expected value to the providers.

2 Replacement Cost Data

For this study, we developed our own cost data so that we could explicitly link these data to data from the U.S. Census. To provide a comprehensive simulated service area, we collected data from two firms and combined them to represent one simulated service area. Firm α provided data on one of its urban service regions containing 78,371 households distributed over 10 service areas, each of which supported a central office node (which may be an end office switch or a remote switching unit). Firm β provided data on one of its rural service regions containing 6490 households distributed over 4 service areas, each of which also supported a central office node[ii].

The two firms gave us facility investment data in the form of 1993 U.S. dollar replacement costs. Other studies that examine incremental access and usage costs have distributed investment costs over the average life of the investment (Mitchell, 1990; Hatfield, 1994). They present the data in a cost per line per month format. Since we did not have usage costs, we limited our study to capital costs only. When we used the estimated cost function in the interactive model (described later), the network costs were levelized over a period of 5 years. Thus, we present the data in the form of total network replacement costs in each service area.[iii] (Molka-Danielsen and Weiss, 1996)

The network elements for basic telephony access that were included in this analysis are:

- **Distribution Link Cost**, which is the material cost of the cable from the host or remote node to the pedestal and the labor cost of installing the cable.
- **Host-to-Remote Cost**, which includes cable, termination, and trunk facilities that connect hosts with simpler remote node.

- **Switch and Software Cost**, which includes the hardware cost of the host or remote node located at a Central Office (CO) site and distributes the software costs according to the number of lines served by this site. The remote nodes that are connected to the host share the remaining software costs. The software costs are distributed from the main switches to the remote nodes because the remote nodes share some of the functionality of the main hosts.
- **Drop Line Cost,** which is the total cost for drop lines from the pedestal to the households. It is based on the number of pairs of lines multiplied by a cost per pair.

The details regarding how the cost data was collected are reported in another paper by (Molka-Danielsen and Weiss, 1996)[iv]. The Total Network Replacement Cost was computed by summing the Distributed Link Cost, Remote to Host Cost, Distributed Switching and Software Cost and Drop Line Cost for each service area. These data are presented in Table 1a[v] and Table 1b.

*********TABLE 1a and TABLE 1a about here

3 Selection of a Market Cost Function

To select a market cost function, we looked for a relation between the total network replacement costs and the number of lines in the service area. A regression line fitting was applied to the replacement cost data from our 14 study service areas. We found that the best fit to the data was obtained using a power curve; the estimated parameters of the power were b_0=36783 and b_1=.5597[vi], which fit with R^2=0.922 and a standard error of 0.188. We let this be the network cost function for the incumbent LEC, *Provider 0*. We computed the average cost per subscriber by summing the total network costs in Table 1a and 1b dividing by the total number of households, yielding an average cost per subscriber of US$902.36 for our combined study region.

To estimate the network cost function for the new entrant (*Provider 1*), we used Reed's study (Reed, 1993). The lowest average cost per subscriber was estimated for the case of an Integrated Network for Telephone and Distributed Video Services with penetration rates of 20% and 40% respectively. For this case, the average cost per subscriber was US$1420. We took the ratio of these two average costs (1.57), and use that to differentiate costs of two providers in our interactive game model. We assume that the incumbent *Provider 0* and the new entrant *Provider 1* have cost functions that are the same shape.

4 Selection of an Access Demand Function

The demand for access for local telephone service is linked to the demand for usage of service. The "service" can be many varieties and bundles of telecommunication services and features. The question "What price should be charged for access and will the demand for access support that price?" has been extensively debated in the (Mitchell and Vogelsang, 1991; Schmalensee, 1981). We are more interested in the question, "What is the demand for access?" We briefly list some studies that have examined the issue of demand for access. The first 5 studies are reviewed and summarized by Taylor and the later two are newer studies done outside the U.S. (Killen, Wynne, and Keogh, 1994; Taylor, 1994; Trotter, 1996). Of the collection, Perl's 1983 study with 1984 revision makes the most extensive use of sociodemographic data, so we chose that model, even though it is 15 years old. We believe that this is still a valid model, as much of the growth of the telephone network has been for second lines, and that the determinants of demand for primary access lines remains basically unchanged.

******Table 2 about here

Perl uses a discrete choice (logit) framework to model a user's choice of whether to have a telephone. This choice is related to monthly cost of service, income and many socio-demographic factors. His model predicts the logarithm of the odds of having a telephone based on exogenous variables. To focus on access, we selected his model for Flat Rate Access Areas, where there is no measured use option available to subscribers. We use Census Tract Household Data from 1990 and matched them to the 14 study service areas (the areas have a population of about 200,000 people in the urban region and 18,000 people in the rural region).

Perl's model estimates *Pr* (the probability of having a telephone) using an intercept (*a*) and coefficients (b_i) for the vector of socio-demographic characteristics, X_i.

$$\ln\left[\frac{\Pr}{1-\Pr}\right] = a + \sum b_i X_i$$

In our use of the model, the values of all of the social-demographic characteristics are preset from Census data except the value for the flat rate price parameter (b_f). The equation becomes:

$$\ln\left[\frac{\Pr}{1-\Pr}\right] = y + b_f X_f$$

where y is a compressed intercept of all the known parameters times their coefficients, plus the Perl intercept. The expression can also be written as:

$$\Pr = \frac{1}{1+e^{-(y+b_f X_f)}}$$

which produces the probability of subscribing for the study service area or the penetration rate. We use this probability function to develop revenue functions for the reaction functions that are used in the interactive game model.

The reaction functions in the game model determine market quantities and market price. To complete the picture, we rewrite Perl's equation to create an inverse demand function:

$$F(q_0 + q_1) = \frac{-1}{b_f\left[\ln\left(\Pr/(\Pr-1)\right) - y\right]}$$

Here, q_0 and q_1 are the number of access lines for *Provider* 0 and *Provider* 1, b_f is the coefficient for flat rate price in the Perl model, y is the compressed intercept, and $F(q_0 + q_1)$ is the price for access. The game model that is discussed in the next section, selects the quantities that will be produced under described market conditions.

5 Description of the Applied Game Model

The game model in our application is an implementation of the Green and Porter model for noncooperative implicit collusion under imperfect information (Bierman and Fernandez, 1993; Green and Porter, 1984). The Green and Porter model represents the strategic behavior of players through repeated games, which can be used to model the long-term relationships of economic or political situations. The model can represent situations where implicit trust can replace explicit contracts to produce higher expected values for all the players involved. New equilibrium outcomes are possible because the players can condition their decision on information that they received from the previous stages of the game. In the Green and Porter model, the publicly observed information is market price.
*****Table 3 about here

The Green and Porter model is well suited to the U.S. local access market because of the present high cost of entry. Also, the product (local access) is homogeneous, and can be offered in one region by two providers. Providers can be collocated, and are assumed to offer facilities-based competition. Information about the industry, such as monthly price for access service, is public information and changes in price are publicly known for residential customers. Providers do not know what facilities their competitors have put in to their networks, so they can not observe one another's outputs (available access lines) directly.

The outcome of the Green and Porter game has also been referred to as a self enforcing cartel agreement, because the game has many equilibria in which the players can choose to cooperate or to not cooperate. Since the cooperative choices can be self-enforcing equilibria, the participants behave as cartel members even though they are oligopolistic competitors. The participants make inferences about the behavior of the other firm by observing the market price. If the market price remains above a certain value, called the trigger price, the firm does not infer a defection from the implicit collusive agreement, but if the market price drops below the trigger price, then retaliation may be warranted.

The players in our model choose a quantity to produce in each period of the game. The Pareto optimal equilibria are limited to a set of strategies in which players choose the Cournot quantities.[vii] The total market quantity produced results in a market price, which serves as information to the firms when they choose a quantity to produce in the next time period.[viii] In cooperative periods, players can select a smaller quantity, which results in a higher discounted profit.

The interactive game begins in a cooperative period. The firms would be best off if they maintained cooperative behavior so they could extract the highest price. However, outside forces, such as a temporary flux in demand, a demand shock, or one of the firms changing their quantity, can cause changes in the market price. If the market price falls below a certain trigger price, then all firms begin to produce at a higher quantity level.

In an *ex post* evaluation of how this game actually evolves, all players correctly infer that their rival chose the cooperative quantity in the last period, and that price is low because of demand shock. Punishment then follows automatically as a self-enforcing reaction to the low realized demand. Because, *ex ante*, players chose an adequate punishment and stuck to it, regardless of whatever the reasons for the price drop, *ex post*, no cheating takes place. It is optimal to punish only because players do not know what caused the drop in price. Under this uncertainty, players cannot abuse the implicit agreement. During punishment periods, players will produce at higher quantities and accept lower prices, because otherwise all firms would have incentive to defect during cooperative periods, and cooperative quantities would never be enacted.

By applying this quantity game we can find the optimal trigger strategies of the two players. That is, we look for the 4-tuple, of trigger price (*tp*), number of punishment periods (T), non-cooperative equilibria quantity (s_i), and the response quantity ($R_{TP}(q_j)$) which is the optimal output of one player in response to the optimal output of the other player. The trigger strategy $(tp, T, s_i, R_{TP}(q_j))$ for player i maximizes the player i's expected present and future discounted value. Similarly, for player j the trigger strategy $(tp, T, s_j, R_{TP}(q_i)t)$ maximizes the player j's expected present and future discounted value.

In our model the expected present discounted value functions for two firms in the market are:

$$VC_i(q_i, q_j) = \frac{\pi_i(s_i, s_j)}{(1-\delta)} + \frac{\pi_i(q_i, q_j) - \pi_i(s_i, s_j)}{(1-\delta) + (\delta - \delta^T)F(x)}$$

$$VCj(q_i, q_j) = \frac{\pi_j(s_i, s_j)}{(1-\delta)} + \frac{\pi_j(q_i, q_j) - \pi_j(s_i, s_j)}{(1-\delta) + (\delta - \delta^T)F(x)}$$

where

$$F(x) = \frac{2}{1 + e^{-\ln 3\gamma}} - 1$$

and

$$\gamma = \left({}^{tp}\!\Big/_{mp(q_i, q_j)^{15}} \right)$$

is the probability that the expected market price (*mp*) is greater than the trigger price (*tp*)[ix]. Other notation is defined as follows.

- s_i and s_j are the strictly noncooperative Cournot quantities that the two firms can choose.
- δ is the discount factor, the way in which firms value future profits.[x]
- T is the number of reversion periods in months
- *tp* is the trigger price in U.S. dollars
- q_i, q_j are the response quantities which optimize the value statements.
- $\pi_i()$ and $\pi_j()$ are profits using the inverse demand function and costs described earlier.

6 Summary and Discussion of Model Results

Our model resolves the discounted value statements presented in the previous section. We search for Pareto optimal values over a range of trigger price values (*tp*) for a given reversion length (*T*). When the Pareto optimal discounted value is found, at a particular *tp*, we then run the model again at that *tp* over a range of different reversion lengths *T*. We then repeat the process testing discounted values at different *tp* using the new *T*. The process is repeated until no higher discounted values can be found. From this procedure we compute the best response quantities, $R_{TP}(q_j)$ and $R_{TP}(q_i)$. We also verify the same response quantities using two different root-finding functions in Mathematica.

Figure 1 is an example of the Pareto optimal response quantities computed for Service Area A. In the figure, the first crosspoint nearest to the axis origin represents the best response quantities for each firm and produces the highest discounted values; it is also the Nash equilibrium. The next crosspoint shows the Cournot quantities, which produce lower discounted values. The furthest crosspoint from the origin shows another suboptimal equilibrium point where selected quantities produce still lower discounted values.

*****Insert Figure 1 about here

6.1 Result 1: The Pareto Optimal Reversion Length Is Long

After examining all 14 service areas, we found that in most of the service areas the discounted values of both firms always increased, but with decreasing increments, as we increased the reversion length (T) in months. The model also showed many sub-optimal tuples where $T<\infty$. In these cases, the collusive quantities still produced higher expected values than Cournot quantities, so selecting these collusive quantities were superior to selecting the Cournot quantities. In game theory, the outcomes produced by the sub-optimal strategies would not be part of the core, because players can do just a little better by punishing forever, however, in practice, firms may not be concerned with the very small difference in expected discounted values that result from using a finite reversion length. In summary, the response curves, of which Figure 1 is an example, are plots of the numerical values of the best quantity responses, demonstrate the point. The quantities at the intersection closest to the origin is the Nash equilibrium in these tringger price strategies, where the discounted expected value for both firms is maximized.

6.2 Result 2: Trigger Prices Are Linked to the Social-demographic Factors

We can also comment about the resulting trigger prices for the 14 service areas. First, the cooperative response prices that can be derived from the computed cooperative response quantities are always greater than the trigger prices. The trigger prices themselves are related to the *y* intercept that was computed for each service area using the Perl model (see Tables 3 and 4). The trigger price is higher in areas that would support greater demand for access. In Table 5 we list the expected market prices that were computed using the quantities in Table 4.

6.4 Result 3: Universal Service Supports Are Desirable

An important result of this study is that universal service supports that would promote greater service penetration in these study areas is desirable since collusive behavior, even if implicit, can occur. Such behavior will adversely affect the number of households receiving service due to higher access prices (see Tables 1a, 1b and 4). We note the total number of households served in each service area using Cournot quantities or cooperative quantities. In Service Area A this is 21765 households using Cournot quantities and 20741 using cooperative quantities and amounts to service penetrations of 82.38% and 78.51% respectively[xi]. The number of households served in a market such as this would be well below current expectations for universal service.

This result supports the notion that some form of subsidy is necessary if social goals are to be achieved, even though this is not an explicit result of the model[xii]. Our data were for capital costs only, and do not include operating costs, suggesting that the actual cost basis is higher than the data reported here suggest. These higher costs could be expected to result in still higher prices, which would in turn reduce service penetration further.

7 Implications for Local Exchange Competition

Our model describes conditions in which collusive behavior in a local exchange market can occur, even though the firms are in fact competitors. We focus our discussion on several important conclusions.

- **Collusive behavior could occur in the study area, but tested conditions are very strict.** Under smaller discount factors δ the reversion length T must grow larger to find collusive outcomes and in some cases there is no collusive outcome (we did not investigate the minimal conditions, the smallest reversion lengths (T), to produce collusive response).
- **Competition is sustainable even in areas where a grim strategy is Pareto optimal.** The grim strategy does not mean that one firm would lose money and therefore exit the market, but instead implies that in order to sustain the collusive price, providers must punish forever. In this case, the only other price the providers can obtain is the Cournot price, which must exceed the long run average. Our market cost data in Table 1a and Table 1b show that an average cost function is downward sloping and approaching a constant marginal cost. Therefore, a price above marginal cost would ensure a price above long run average costs. Since we know the Cournot price is above marginal cost we can conclude the firms would not go out of business by not colluding, they would just earn lower economic profits.
- **Low service penetration is possible without universal service supports**. This study indicates that the percentage of households that would receive local exchange access under

market conditions is well below the expected penetration rates for universal access. Along with lower penetration rates, the price for access would be much higher than typical local access tariffs in the U.S.

This result supports proposed policies for universal service subsidies. More specifically, this model uses a demand for access model that represents a study area's socio-demographic features. The socio-demographic *y* intercept for all the 14 study areas reflects a strong demand for telephone access. The model shows areas with low demand for access would have lower trigger prices, and that collusion could more easily occur in those areas. From this observation we can conclude that universal service support would be particularly important in areas with lower demand for access.

******Insert Table 4 and 5 about here.

- **Our model cannot be applied when there are dynamic changes in the demand for access**. The application of our trigger price model is appropriate for the scope of this analysis because the demand for access has been growing at a steady and predictable rate for the past decade. We start with an initial state of demand and make projections forward based on that initial state. New technology, such as wireless access, can introduce lower cost access networks and substitute products and services.

We conjecture that non-facilities-based entry into the local access market creates less clear market boundaries between service markets. Since access services can be more easily combined with different bundles of services, it becomes difficult to estimate demand for access alone. We also speculate that the policies for allocating loop costs based on pre-subscribed lines would be less effective. IXCs could pass on better savings to partner CLECs where partnerships may only evolve in higher demand areas. Again, market pressure that will lower consumer prices we predict would not arise easily. This points again to the present applicability of our model and the need for universal access assistance for consumers.

References

H. Scott Bierman and Luis Fernandez. Game Theory with Economic Applications. Addison-Wesley Publishing Company, New York, 1993.

Edward Green and Robert Porter. ``Noncooperative collusion under imperfect price information." Econometrica, 52:87-100, 1984.

Hatfield Associates Inc. The Cost of Basic Universal Service. Prepared for MCI Communications Corporation, July 1994.

L. Killen L. Wynne, G. Keogh. Estimation of Demand for Residential Telephone Connections in Ireland. Technical report, Dublin City University, Dublin, Ireland, 1994. CA-0594.

Bridger M. Mitchell. Incremental Capital Costs of Telephone Access and Local Use. Number R-3909-ICTF. The RAND Corporation, July 1990.

Bridger M. Mitchell and Ingo Vogelsang. Telecommunications Pricing: Theory and Practice. Cambridge University Press, 1991.

Judith Molka-Danielsen and Martin B.H. Weiss. "Local telephony access costs: Empirical cost functions for rural and urban areas". In Proceedings of the 4th International Conference on Telecommunications Systems, Modeling and Analysis, March 1996, pp. 21-24.

Lewis J. Perl. Residential Demand for Telephone Service. Technical report, NERA Report, December 1983.

Lewis J. Perl. Revision to NERA's Residential Demand for Telephone Service 1983. Technical report, NERA Report, April 1984.

Robert Porter. ”Optimal cartel trigger price strategies”. Journal of Economic Theory, 29:313-338, 1983.

David Reed. ”The prospects for competition in the subscriber loop: The fiber-to-the-neighborhood approach”. In The Twenty-First Annual Telecommunications Research Policy Conference, Solomons Is., Maryland USA, 1993.

Bell Communications Research. BOC Notes on LEC Networks. Number SR-TSV-002275, Issue 2. April 1994.

Steven C. Salop. ”Practices that (credibly) facilitate oligopoly coordination”. In Joseph~E. Stiglitz and G. Frank Mathewson, editors, New Developments in Analysis of Market Structure, pages 265-290. MIT Press, Cambridge MA, 1986.

Richard Schmalensee. ”Monopolistic two-part tariff arrangements”. Bell Journal of Economics, pages 445-466, 1981.

Lester D. Taylor. Telecommunications Demand in Theory and Practice. Kluwer Academic Publishers, Dordrecht, Nederlands, 1994.

Stephen Trotter. ”The demand for telephone services”. Applied Economics, 28:175-184, 1996.

Co	Distribution Link Cost	Drop Line Cost	Remote to Host Cost	Switch & Soft Cost	Total Net Cost	Number Lines	Number Households
A	1932718	3516500	0	5096443	10545661	28132	26419
B	1020992	3622000	0	4324899	8967891	28976	15585
C	616415	3950000	37511	8011259	12615185	31600	16024
D	1268762	2192000	14206	5376567	8851535	17536	9985
E	533074	247468	28412	1688932	2486168	1866	1372
F	105497	293750	20904	2024818	2444969	2350	346
G	394879	528000	74463	3694285	4691627	4223	3539
H	1144764	798750	87079	5878378	7908971	6390	1740
I	175991	638750	36562	3540422	4391725	5110	1713
J	147743	239750	40720	2072316	2500529	1917	1648

Table 1a. Firm α: Urban Area Details of Costs

Co	Distribution Link Cost	Drop Line Cost	Remote to Host Cost	Switch & Soft Cost	Total Net Cost	Number Lines	Number Households
K	1891368	425125	NA	1587500	3904011	3401	2267
L	1350990	307500	NA	1362500	3020990	2460	1640
M	1188871	271000	NA	845000	2304871	2168	1445
N	972713	213375	NA	755000	1941088	1707	1138

Table 1b. Firm β: Rural Area Details of Costs

Study Author	Dependent Variable	Price Elasticity on basic access	Type of Data
Alleman	main-stations	-.17	cross-section US cities
Feldman	main-stations	-.05	cross-section US states
Perl	telephone-availability	-.08	cross-section US households
Rash	main-stations	-.11	time-series, Ontario-Quebec
Waverman	main-stations	-.12	time-series, Ontario-Quebec
Wynne	residential-lines	n.a.	time-series, Dublin Ireland
Trotter	residential-lines	-.1	cross-section, Hull UK

Table 2. Summary of Access Demand Studies

SA	y	SA	y
A	4.730	H	4.714
B	6.330	I	4.779
C	5.411	J	4.506
D	5.557	K	5.263
E	5.654	L	4.960
F	5.153	M	5.067
G	5.872	N	4.682

Table 3. Intercepts for different service areas

SA	T	tp	Cournot Q0	response Q0	Cournot Q1	response Q1	discounted value P0	discounted value P1
A	1227	53	11196	10679	10569	10062	324329671	273652415
B	1697	73	6981	6831	6639	6491	281849026	243035603
C	5000	61	6996	6760	6577	6345	231823833	192891963
D	5000	63	4398	4263	4075	3943	145638897	115581710
E	5000	68	636	617	492	474	16447051	6079643
F	5000	66	132	125	132	125	888902	888902
G	1245	69	1596	1552	1408	1365	51235412	34118957
H	5000	57	781	743	573	539	16017613	4297454
I	5000	58	771	733	568	534	16088010	4461609
J	5000	55	738	700	516	482	13990323	2548811
K	5000	62	1019	982	832	797	26044054	12617863
L	5000	60	660	632	477	452	14047019	3361014
M	5000	61	745	714	564	535	16846068	5481925
N	1860	59	531	503	329	305	9794068	112343

Table 4. Trigger Strategies, T=5000

SA	Cournot price	cooperative response price
A	67.39	72.61
B	92.89	96.34
C	78.22	82.66
D	81.04	85.16
E	87.16	90.85
F	84.21	88.70
G	87.67	91.47
H	73.12	77.89
I	74.07	78.96
J	70.79	75.59
K	79.71	83.92
L	87.62	90.75
M	59.25	67.97
N	75.10	80.04

Table 5. Expected Market Price, in monthly U.S. dollars

Endnotes:

---

[i] Special thanks to Stéphane Pallage, of CREFE and University of Quebec at Montreal, Department of Ecomonics, for his critical review of the described game.

[ii] For both firms, the central office node can be either a central office switch or a remote access node. The remote nodes can provide single line and single party access to the first point of switching in the local exchange network. This definition of a service area is similar to the definition that was used by Bellcore for defining Carrier Service Areas (CSA). In the Bellcore definition, the CSA can be served by Digital Loop Carriers (DLC) which use T1 remote terminals to connect subscriber loops to central office equipment (Bellcore, 1994). In our study we also include the Remote to Host cost, but the technology can be other than DLC systems.

[iii] The analysis is presented so that the identity and service locations of Firm α and Firm β remain anonymous at their request.

[iv] The Remote to Host Cost includes only the trunk costs between the Remote Switch Concentrator (RSC) or a Remote Line Concentrator (RLC) and an end office switch. For all of the service areas that use RSC or RLC, these costs are placed under Switching Costs. The Remote to Host trunk costs for the last four service areas (K-N) could not be separated from the Distribution Link Costs, so they are included under that column.

[v] Note that service area F is an outlier in the data, in that 2350 lines were constructed for 346 households. Subsequent research uncovered a prison in this service area, which is in part responsible for the overbuild. We have re-tested the model without this service area, and the results are essentially unchanged. We have been unable to go back to the sources of the data to investigate this (and some other less obvious) anomolies further.

[vi] For example, T is the total network cost and L is the number of lines built. Using numbers from Service Area A, T(observed) is 10,545,661 U.S. dollars and the variable L is 28132 access lines. If the independent variable (L = 28132) is applied to the equation we see T(power) is estimated to be US$11,372,780.

[vii] An equilibrium is Pareto optimal if no other equilibria is Pareto superior to it. An equilibrium is Pareto superior to another if none of the players is worse off in the new equilibrium and one at least is better off than in the other.

[viii] A quantity game rather than a price game was selected because once the quantity is produced (number of lines), the provider cannot change the price asked for, but must accept the market price. Distribution of profits cannot be renegotiated. This is a condition in a game where no explicit communication is suppose to take place.

[ix] We take this probability function from Bierman and Fernandez (Bierman and Fernandez, 1993). Our results are not particularly sensitive to this function - we tried several and the basic results still hold.

[x] We selected a high discount value to help demonstrate potential cooperative outcomes. That is .998. In reality, firms in the local access market may make business decision with a look ahead of only 5, 7, 13, or 18 years. This would have a discount factor of .35, .50, .70, or .75 respectively. An alternative selection of delta would change the model's outcomes.

[xi] We assume that all costs for the network are recovered out of the access cost. In practice, a portion of the costs may be recovered from usage based costs, which would improve the service penetration.

[xii] This issue is actually a good bit more complex. For example, we do not compute potential revenues from other services produced over the same infrastructure.